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Five-brane thresholds and membrane instantons in four-dimensional heterotic M-theory

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Abstract

The effective four-dimensional supergravity of M-theory compactified on the orbifold S^1/Z_2 and a Calabi–Yau threefold includes in general moduli supermultiplets describing massless modes of five-branes. For each brane, one of these fields corresponds to fluctuations along the interval. The five-brane also leads to modifications of the anomaly-cancelling terms in the eleven-dimensional theory, including gauge contributions located on their world-volumes. We obtain the interactions of the brane "interval modulus" predicted by these five-brane-induced anomaly-cancelling terms and we construct their effective supergravity description. In the condensed phase, these interaction terms generate an effective non-perturbative superpotential which can also be interpreted as instanton effects of open membranes stretching between five-branes and the S^1/Z_2 fixed hyperplanes. Aspects of the vacuum structure of the effective supergravity are also briefly discussed.

1. Introduction

Heterotic $E_8 \times E_8$ strings compactified to four dimensions on a six-dimensional space K_6 are also described by M-theory compactified on $K_7 \equiv S^1/Z_2 \times K_6$ [1,2]. In particular, it is straightforward to verify [3–6] that the effective $\mathcal{N}_4 = 1$ supergravity found in Calabi–Yau or orbifold compactifications of perturbative heterotic strings [7,8] is reproduced by brane-free M-theory configurations with compact space K_7. A novelty of the M-theory approach lies in the possibility to concretely analyse physical effects of non-perturbative brane configurations. In the

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low-energy effective supergravity approximation, configurations with five-branes and/or membranes (two-branes) [1,9,10] of compactified M-theory can be studied from simple modifications of the field equations predicted by eleven-dimensional supergravity [11].

An obvious distinction in the nature of five-brane and membrane effects follows from the alignement conditions applying to their respective world-volumes if one requires that the configuration admits (exact or spontaneously broken) $\mathcal{N}_4 = 1$ supersymmetry (four supercharges). Each five-brane world-volume is the product of four-dimensional space–time and a holomorphic two-cycle in the Calabi–Yau threefold and conditions apply on the respective cycles of pairs of world-volumes [1,10]. Five-brane massless excitations [12], which belong to six-dimensional chiral supersymmetry multiplets expanded in modes of the two-cycle, lead then to new four-dimensional fields to be included in the effective supergravity description. Some of these modes do not depend on the detail of the Calabi–Yau geometry: the five-brane modulus describing fluctuations along the S^1/Z_2 direction, the two-index antisymmetric tensor $\hat{B}_{\mu\nu}$ with self-dual field strength and their fermionic $\mathcal{N}_4 = 1$ partner. These states can be assembled either in a chiral supermultiplet which we will call $\hat{S}$ or, in a dual version, in a linear multiplet. The effective supergravity for this "universal five-brane modulus" supermultiplet has been studied in Ref. [13] (see also Ref. [14]).[1] Firstly, the Kähler potential of the theory with this new superfield has been obtained and the absence of direct contributions to the (perturbative) superpotential has been demonstrated. Secondly, on the basis of the four-dimensional superspace structure only, the possible appearance of new threshold corrections has been emphasized.

In contrast, open membrane euclidean world-volumes include the S^1/Z_2 direction and a cycle in K_6 [10,17]. They stretch between the S^1/Z_2 fixed planes, or between a fixed plane and a five-brane, or between pairs of five-branes. Their effects in the four-dimensional effective supergravity are then localized in space–time, they can be viewed as instanton-like corrections to the interaction Lagrangian. While open membrane stretching between the fixed hyperplanes correspond in the string approach to world-sheet instantons, membranes ending on a five-brane describe forces acting on this brane. Their contributions to the effective supergravity are then expected to lead to new (non-derivative since the world-volume includes S^1/Z_2) interactions involving the five-brane modulus.

The corrections to the effective four-dimensional supergravity induced by the various types of membranes have been studied in Refs. [14,18,19]. They were found to contribute to the chiral F-density part of the Lagrangian density, in the form of a non-perturbative superpotential. Specifically, an interaction bilinear in the five-brane fermion in superfield $\hat{S}$ has been computed in the four-dimensional background with the five-brane and open membranes ending on it. The resulting non-perturbative superpotential shows an exponential dependence on the five-brane universal modulus typical of instanton calculus. To isolate the membrane contributions from other possible non-perturbative sources, a specific regime is chosen.[2] As a consequence, even if the instanton calculation clearly establishes the existence of an exponential dependence on $\hat{S}$, it does not allow to infer how this exponential term would combine with other non-perturbative contributions which, like gauge instantons, are expected as well.[3]

1 And, as a function of a non-trivial background value of the five-brane modulus, Refs. [15,16].

2 For instance, Moore, Peradze and Saulina [14] select a regime where "open membrane instanton effects are the leading source of non-perturbative effects".

3 Writing the complete non-perturbative superpotential as a sum of contributions, as for instance in Ref. [14], is an assumption which needs to be justified.