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The chirally rotated Schrödinger functional with Wilson fermions and automatic O(a) improvement

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Abstract

A modified formulation of the Schrödinger functional (SF) is proposed. In the continuum it is related to the standard SF by a non-singlet chiral field rotation and therefore referred to as the chirally rotated SF (χSF). On the lattice with Wilson fermions the relation is not exact, suggesting some interesting tests of universality. The main advantage of the χSF consists in its compatibility with the mechanism of automatic O(a) improvement. In this paper the basic set-up is introduced and discussed. Chirally rotated SF boundary conditions are implemented on the lattice using an orbifold construction. The lattice symmetries imply a list of counterterms, which determine how the action and the basic fermionic two-point functions are renormalised and O(a) improved. As with the standard SF, a logarithmically divergent boundary counterterm leads to a multiplicative renormalisation of the quark boundary fields. In addition, a finite dimension 3 boundary counterterm must be tuned in order to preserve the chirally rotated boundary conditions in the interacting theory. Once this is achieved, O(a) effects originating from the bulk action or from insertions of composite operators in the bulk can be avoided by the mechanism of automatic O(a) improvement. The remaining O(a) effects arise from the boundaries and can be cancelled by tuning a couple of O(a) boundary counterterms. The general results are illustrated in the free theory where the Sheikholeslami–Wohlert term is shown to affect correlation functions only at O(a^2), irrespective of its coefficient.

Keywords: Lattice QCD; Schrödinger functional; Dirichlet boundary conditions; O(a) improvement

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1. Introduction

The Schrödinger functional (SF) [1–3] has become a general tool to address non-perturbative renormalisation problems in lattice QCD [4][1] and is now even used in candidate theories which might describe a strongly interacting electroweak sector [13–17]. Renormalisation schemes based on the SF are gauge invariant, quark mass independent through renormalisation in the chiral limit and suitable for evaluation by both Monte Carlo and perturbative methods. Moreover, as the finite space–time volume is used to set the renormalisation scale, recursive finite size techniques can be applied to bridge large scale differences, thereby avoiding to resolve widely different scales on a single lattice. As systematic errors in large scale lattice simulations are becoming smaller, the need for a better controlled continuum limit will likely stir more interest in SF schemes, and it is therefore necessary to address its remaining shortcomings.

A first problem consists in the difficulty to implement the SF boundary conditions for fermion regularisations other than Wilson fermions. This has been largely solved, and formulations of the SF with staggered and Ginsparg–Wilson-type fermions have been given in [18–20] and [21–25], respectively.

The main problem with SF schemes is the presence of lattice artefacts which are linear in the lattice spacing a. Some of these $\mathrm{O}(a)$ effects are caused by the mere presence of the Euclidean time boundaries, together with local Dirichlet conditions for the fields. Such $\mathrm{O}(a)$ boundary effects will be present with any regularisation, since they do not arise from the breaking of a continuum symmetry. In practical applications, these effects are cancelled by adding a couple of boundary counterterms to the action, with coefficients determined perturbatively up to two-loop order [26,27]. While this is often sufficient in practice, a non-perturbative determination of such boundary counterterms would be desirable and is indeed conceivable.

With Wilson quarks there is a second category of $\mathrm{O}(a)$ effects which are cancelled by the usual $\mathrm{O}(a)$ counterterms to the Wilson quark action and the composite fields which appear in the correlation functions. At first sight it may be surprising that these standard $\mathrm{O}(a)$ counterterms are required at all, given that massless Wilson quarks in a finite volume enjoy the property of automatic $\mathrm{O}(a)$ improvement [28]. However, the argument for automatic $\mathrm{O}(a)$ improvement relies on a discrete chiral symmetry which is expected to be recovered in the continuum limit. The argument fails as the standard SF boundary conditions break chiral symmetry, so that observables cannot be classified as either even or odd under this symmetry [29,30].

In this paper a modified definition of the Schrödinger functional for Wilson-type quarks is introduced, which is suitable for QCD with an even number of quark flavours, and which is compatible with automatic $\mathrm{O}(a)$ improvement. In the continuum limit this modified SF is related to the standard SF by a chiral field rotation and is therefore referred to as the chirally rotated SF (χSF). A first account of this work has been given a while ago in Refs. [29,30]. The paper is organised as follows: in Section 2 the argument of automatic $\mathrm{O}(a)$ improvement is reviewed and the reason why it fails in the presence of SF boundary conditions. Possible modifications of the SF boundary conditions which may restore this argument are discussed next. Concentrating on the solution provided by the chirally rotated SF, some continuum considerations are made in Section 3, and its lattice implementation through an orbifold construction is explained in Section 4. The lattice symmetries imply the counterterm structure relevant for renormalisation and $\mathrm{O}(a)$ improvement (Section 5). In Section 6, the renormalisation constants and boundary $\mathrm{O}(a)$ improvement coefficients are determined to tree-level of perturbation theory, and automatic bulk

[1] See [5] for early references and [6–12] for a selection of more recent applications and further references.