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The rational parts of one-loop QCD amplitudes II: The five-gluon case

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Abstract

The rational parts of 5-gluon one-loop amplitudes are computed by using the newly developed method for computing the rational parts directly from Feynman integrals. We found complete agreement with the previously well-known results of Bern, Dixon and Kosower obtained by using the string theory method. Intermediate results for some combinations of Feynman diagrams are presented in order to show the efficiency of the method and the local cancellation between different contributions.

1. Introduction

In a previous paper [1], we developed a method for computing the rational parts of one-loop amplitudes directly from Feynman integrals. The purpose of this paper is to apply this method to compute the rational parts of 5-gluon one-loop amplitudes. The result agrees with the well-known result of Bern et al. [2] obtained first by using string-inspired methods. (Other 5-parton amplitudes in massless QCD were later computed by using either standard Feynman diagrammatic technique [3] or supersymmetric decomposition and perturbative unitarity [4].)

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The computation of the multi-particle one-loop amplitudes in QCD is a very difficult problem. Even for 4-parton amplitudes the computation is quite non-trivial [5]. For 5-gluon amplitudes a new method was developed [6] by using string theory.

The constant effort to calculate multi-leg one-loop amplitudes lies in the application to the forthcoming experimental program at CERN's large hadron collider (LHC), as there are lots of processes with many particles as final states [7]. We refer the reader to [1] for a discussion and extensive references for the recent efforts in computing the multi-leg one-loop amplitudes and the recent developments inspired by twistor string theory [8,9].

In order to compute multi-leg one-loop amplitudes in QCD, it is a good strategy [10] to decompose the QCD amplitudes into simpler ones by using the supersymmetric decomposition:

$$A^{\mathrm{QCD}} = A^{N=4} - 4A^{N=1\ \mathrm{chiral}} + A^{N=0\ \mathrm{or\ scalar}}, \tag{1}$$

where A^{QCD} denotes an amplitude with only a gluon circulating in the loop, $A^{N=4,1}$ have the full $N = 4, 1$ multiplets circulating in the loop, and $A^{N=0}$ has only a complex scalar in the loop.

By using the general properties of the one-loop amplitudes, Bern, Dunbar, Dixon and Kosower proved that the supersymmetric amplitudes $A^{N=4,1}$ are completely determined by 4-dimensional unitarity [10], i.e. the amplitudes are completely cut-constructible and the rational parts are vanishing (see [1,10] for more detail explanation). For MHV helicity configurations, explicit results were obtained for $A^{N=4}$ in [10]. The recent development of using MHV vertices to compute one-loop amplitudes leads to many new results for the cut-constructible part [11–22]. In particular, Bedford, Brandhuber, Spence and Travaglini [11,15] applied the MHV vertices to one-loop calculations. Britto, Buchbinder, Cachazo, Feng and Mastrolia [20–22] developed an efficient technique for evaluating the rational coefficients in an expansion of the one-loop amplitudes in terms of scalar box, triangle and bubble integrals (the cut-constructible part, see [1] for details). By using their technique, it is much easier to calculate the coefficients of box integrals without doing any integration. Recently, Britto, Feng and Mastrolia completed the computation of the cut-constructible terms for all the 6-gluon helicity amplitudes [22].

In order to complete the QCD calculation for the 6-gluon helicity amplitudes, the remaining challenge is to compute the rational parts of the helicity amplitudes with scalars circulating in the loop. In general, we need an efficient and powerful method to compute the rational part of any amplitude.

As we reviewed in [1], there are various approaches [23–28] to compute the rational part. In particular, Bern, Dixon and Kosower [27,28] developed the bootstrap recursive approach which has lead to quite general results [29–31]. In this paper we will use the approach as developed in [1] and apply it to compute the rational parts of the 5-gluon one-loop amplitudes. In another paper [32] we will compute the rational parts of 6-gluon amplitudes in QCD (see also [31]) which are the last missing pieces for the complete partial helicity amplitudes of the 6-gluon one-loop QCD amplitude.

This paper is organized as follows: in Section 2, we recall briefly the Feynman diagrams and the Feynman rules, tailored for our computation of the 5-gluon amplitudes. Some simple reduction formulas are recalled briefly in Section 3. In Section 4 we summarize all the integral formulas we will use in this paper. Then the following 2 sections present the results for the rational parts of the two independent MHV helicity configurations.