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# Two-loop vertices in quantum field theory: Infrared and collinear divergent configurations

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**Abstract**

A comprehensive study is performed of two-loop Feynman diagrams with three external legs which, due to the exchange of massless gauge-bosons, give raise to infrared and collinear divergencies. Their relevance in assembling realistic computations of next-to-next-to-leading corrections to physical observables is emphasized. A classification of infrared singular configurations, based on solutions of Landau equations, is introduced. Algorithms for the numerical evaluation of the residues of the infrared poles and of the infrared finite parts of diagrams are introduced and discussed within the scheme of dimensional regularization. Integral representations of Feynman diagrams which form a generalization of Nielsen–Goncharov polylogarithms are introduced and their numerical evaluation discussed. Numerical results are shown for all different families of multi-scale, two-loop, three-point infrared divergent diagrams and successful comparisons with analytical results, whenever available, are performed. Part of these results has already been included in a recent evaluation of electroweak pseudo-observables at the two-loop level.


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## 1. Introduction

This paper belongs to a series devoted to numerical evaluation of the multi-loop, multi-leg Feynman diagrams that appear in any renormalizable quantum field theory. In [1] (hereafter I)

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the general strategy has been designed and in [2] (hereafter II) a complete list of results has been derived for two-loop functions with two external legs, including their infrared divergent on-shell derivatives. Results for one-loop multi-leg diagrams have been shown in [3] and additional material can be found in [4]. Two-loop three-point functions for infrared convergent configurations have been considered in [5] (hereafter III), two-loop tensor integrals in [6].

Many mass scales appear in the computation of physical observables within the Standard Model, generating serious difficulties for the familiar analytical approach. Our purpose is to overcome these problems through a numerical approach. The application of our techniques has recently contributed to the evaluation of the two-loop fermionic correction to the effective electroweak mixing angle and of the full Higgs-mass dependence of the bosonic ones [7].

The approach described in [1] is primarily intended for evaluation of multi-loop diagrams with internal massive lines. However, QED and QCD are integral part of any realistic calculation and they lead to infrared singularities. Therefore, any method aimed to a numerical evaluation of diagrams must be able to handle the infrared problem and infrared/collinear configurations should be treatable within the same class of algorithms used for the non-infrared cases or within some simple extension of the latter.

For one-loop diagrams we have seen that our methods allow us to extract the infrared pole in dimensional regularization with a residue and a finite part that can be treated numerically [3]. The procedure has been extended in II to cover the on-shell derivative of two-point functions which are needed in the treatment of external legs.

It is the purpose of this paper to extend the study of infrared divergencies to two-loop three-point functions. All diagrams are computed within the scheme of dimensional regularization [8] with space–time dimensionality $n = 4 - \epsilon$. Each loop in a diagram contributes at most one soft (zero gauge-boson mass) and one collinear (for zero fermion mass) $1/\epsilon$ term but the global order of the pole at $\epsilon = 0$ can be greater than two due to simultaneous occurrence of ultraviolet poles which are removed by the introduction of counter-terms.

To accomplish our goals we need an automatized procedure for handling infrared (and collinear) configurations: Landau equations [9] represent the proper tool since a necessary condition for the presence of infrared divergencies is that the Landau equations are fulfilled. Therefore, for each topology we build individual diagrams by filling all the lines with the line content of the theory, disregarding those configurations with vertex content not allowed by the theory itself. The generated result is examined and Landau equations studied for those diagrams that contain massless gauge-boson: if they are fulfilled then we have an infrared divergent configuration. The residue of the infrared pole(s) and the corresponding infrared finite part are then computed numerically.

This part of the procedure is relatively easy while the difficult task is connected to the numerical evaluation of residues and of finite parts. They will be given in terms of multi-dimensional integrals over Feynman parameters with integrands that are not positive defined and, according to our strategy, their evaluation requires introduction of smoothness algorithms.

Smoothness requires that, after suitable manipulations, the kernel in the integral representation and its first $N$ derivatives be continuous functions and, ideally, $N$ should be as large as possible. However, in most of the cases we will be satisfied with absolute convergence, e.g., logarithmic singularities of the kernel. This is particularly true when the large number of terms required by obtaining continuous derivatives of higher order leads to large numerical cancellations.

There is a general approach for extracting infrared poles which goes under the name of sector decomposition [10]. We have examined this technique which, despite its great intrinsic possibilities, has its own problems: to name one it has been applied (so far) mainly to unphysical