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Improved perturbation method and its application to the IIB matrix model

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Abstract

We present a new scheme for extracting approximate values in "the improved perturbation method", which is a sort of resummation technique capable of evaluating a series outside the radius of convergence. We employ the distribution profile of the series that is weighted by nth-order derivatives with respect to the artificially introduced parameters. By those weightings the distribution becomes more sensitive to the "plateau" structure in which the consistency condition of the method is satisfied. The scheme works effectively even in such cases that the system involves many parameters. We also propose that this scheme has to be applied to each observable separately and be analyzed comprehensively.

We apply this scheme to the analysis of the IIB matrix model by the improved perturbation method obtained up to eighth order of perturbation in the former works. We consider here the possibility of spontaneous breakdown of Lorentz symmetry, and evaluate the free energy and the anisotropy of space–time extent. In the present analysis, we find an SO(10)-symmetric vacuum besides the SO(4)- and SO(7)-symmetric vacua that have been observed. It is also found that there are two distinct SO(4)-symmetric vacua that have almost the same value of free energy but the extent of space–time is different. From the approximate values of free energy, we conclude that the SO(4)-symmetric vacua are most preferred among those three types of vacua.

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1. Introduction

String theory is the unique theory that contains massless spin-two particles, i.e., gravitons [1], and thus it is considered to provide an unified microscopic description of the universe including gravitational interactions. For this reason the string theory has been subjected to intensive studies. However, it is recognized that the perturbative string theory fails to single out our universe as the unique vacuum of the theory [2]. Therefore we are forced to pursue non-perturbative formulations. The IIB matrix model (also called the IKKT matrix model) is proposed as a constructive formulation of the superstring theory [3,4].

A significant feature of the IIB matrix model is that the space–time itself is expressed by the eigenvalue distribution of 10 bosonic matrices, and thus it is treated as a dynamical variable of the model. The origin of our four-dimensional space–time can be argued in the context of the IIB matrix model as a spontaneous breakdown of Lorentz symmetry. In this regard, we have to understand non-perturbative properties of the model.

The mechanism of the spontaneous breakdown of Lorentz symmetry in reduced matrix models has been examined in various approaches. It has been recognized from those works that the fermionic part of the action plays a crucial role [5–11].

Unveiling dynamical aspects of the model is, in general, quite a difficult problem. The Monte Carlo method is a powerful tool for exploring such non-perturbative properties of a model. However, it is not applicable (or at least quite difficult to apply) to the IIB matrix model due to complex phase of the action derived from the fermionic part.[1] The improved perturbation method (also called the Gaussian expansion method) is an alternative approach. It is considered as a sort of variational method [13–18]. It has been successfully applied to various models [19–22], and applications to matrix models were done in Refs. [23–25].

The application to the IIB matrix model was first achieved in Ref. [26] in which various patterns of symmetry breaking that preserve SO(d) subgroup of the original ten-dimensional rotational symmetry (ansatz) were examined, and as a conclusion, four-dimensional universe is the most preferred among them based on the comparison of free energy. The Gaussian expansion method was reformulated as an improvement of perturbative series expansion in Ref. [27]. The improved Taylor expansion (ITE), as it is referred, opened a way toward more general applications that incorporate quadratic and other types of interactions. The ITE prescription was employed for the IIB matrix model up to fifth order of perturbation in Ref. [27]. It has been proceeded to even higher orders and extended ansatz [28–30]. The mechanism of the spontaneous symmetry breakdown is further examined in a simplified model via the Gaussian expansion method in Ref. [31].

In the present paper, we investigate the non-perturbative solutions of the IIB matrix model by this technique (which we call "the improved perturbation method" here). We focus on the possibility that the original SO(10) symmetry of the IIB matrix model may be spontaneously broken to result in our universe which spreads in four directions and has SO(4) rotational symmetry.[2] In such cases it is important to see that the method is applicable to the models that exhibit phase transitions. In the former work [22], we applied this method to the Ising model and found that the improved perturbation method extracts the information of the ordered phase from an expansion about the vacuum in the disordered phase. It is also observed that an unstable vacuum is identi-

[1] A novel technique called the factorization method is proposed to resolve the complex action problem in the Monte Carlo simulations [11,12].

[2] We perform the Wick rotation to the IIB matrix model and discuss in Euclidean space–time.