Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Nuclear Physics B 744 [FS] (2006) 312–329
NUCLEAR PHYSICS B

T–Q relation and exact solution for the XYZ chain with general non-diagonal boundary terms

Wen-Li Yang [a,b,*], Yao-Zhong Zhang [b]

[a] Institute of Modern Physics, Northwest University, Xian 710069, PR China
[b] Department of Mathematics, University of Queensland, Brisbane, QLD 4072, Australia

Received 22 February 2006; received in revised form 11 March 2006; accepted 16 March 2006

Available online 6 April 2006

Abstract

We propose that the Baxter's Q-operator for the quantum XYZ spin chain with open boundary conditions is given by the $j \to \infty$ limit of the corresponding transfer matrix with spin-j (i.e., $(2j+1)$-dimensional) auxiliary space. The associated T–Q relation is derived from the fusion hierarchy of the model. We use this relation to determine the Bethe Ansatz solution of the eigenvalues of the fundamental transfer matrix. The solution yields the complete spectrum of the Hamiltonian.
© 2006 Elsevier B.V. All rights reserved.

PACS: 75.10.Pq; 04.20.Jb; 05.50.+q

Keywords: Spin chain; Reflection equation; Bethe Ansatz; Q-operator; Fusion hierarchy

1. Introduction

In this paper we are interested in the exact solution of the quantum XYZ quantum spin chain with most general non-diagonal boundary terms, defined by the Hamiltonian

$$H = \sum_{j=1}^{N-1} \left(J_x \sigma_j^x \sigma_{j+1}^x + J_y \sigma_j^y \sigma_{j+1}^y + J_z \sigma_j^z \sigma_{j+1}^z \right) + h_x^{(-)} \sigma_1^x + h_y^{(-)} \sigma_1^y + h_z^{(-)} \sigma_1^z + h_x^{(+)} \sigma_N^x + h_y^{(+)} \sigma_N^y + h_z^{(+)} \sigma_N^z, \tag{1.1}$$

* Corresponding author.
E-mail address: wenli@maths.uq.edu.au (W.-L. Yang).

0550-3213/$ – see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.nuclphysb.2006.03.025

where σ^x, σ^y, σ^z are the usual Pauli matrices, N is the number of spins, the bulk coupling constants J_x, J_y, J_z are related to the crossing parameter η and modulus parameter τ by the following relations,

$$J_x = \frac{e^{i\pi\eta}\sigma(\eta+\frac{\tau}{2})}{\sigma(\frac{\tau}{2})}, \qquad J_y = \frac{e^{i\pi\eta}\sigma(\eta+\frac{1}{2}+\frac{\tau}{2})}{\sigma(\frac{1}{2}+\frac{\tau}{2})}, \qquad J_z = \frac{\sigma(\eta+\frac{1}{2})}{\sigma(\frac{1}{2})},$$

and the components of boundary magnetic fields associated with the left and right boundaries are given by

$$h_x^{(\mp)} = \pm e^{-i\pi(\sum_{l=1}^{3}\alpha_l^{(\mp)}-\frac{\tau}{2})}\frac{\sigma(\eta)}{\sigma(\frac{\tau}{2})}\prod_{l=1}^{3}\frac{\sigma(\alpha_l^{(\mp)}-\frac{\tau}{2})}{\sigma(\alpha_l^{(\mp)})}, \qquad h_z^{(\mp)} = \pm\frac{\sigma(\eta)}{\sigma(\frac{1}{2})}\prod_{l=1}^{3}\frac{\sigma(\alpha_l^{(\mp)}-\frac{1}{2})}{\sigma(\alpha_l^{(\mp)})},$$

$$h_y^{(\mp)} = \pm e^{-i\pi(\sum_{l=1}^{3}\alpha_l^{(\mp)}-\frac{1}{2}-\frac{\tau}{2})}\frac{\sigma(\eta)}{\sigma(\frac{1}{2}+\frac{\tau}{2})}\prod_{l=1}^{3}\frac{\sigma(\alpha_l^{(\mp)}-\frac{1}{2}-\frac{\tau}{2})}{\sigma(\alpha_l^{(\mp)})}. \tag{1.2}$$

Here $\sigma(u)$ is the σ-function defined in the next section and $\{\alpha_l^{(\mp)}\}$ are free boundary parameters which specify the boundary coupling (equivalently, the boundary magnetic fields).

In solving the closed XYZ chain (with periodic boundary condition), Baxter constructed a Q-operator [1], which has now been proved to be a fundamental object in the theory of exactly solvable models [2]. However, Baxter's original construction of the Q-operator was ad hoc and its connection with the quantum inverse scattering method was not clear. It was later argued [3] that the Q-operator for the XXZ spin chain with periodic boundary condition may be constructed from the $U_q(\widehat{sl_2})$ universal L-operator whose associated auxiliary space is taken as an infinite-dimensional q-oscillator representation of $U_q(\widehat{sl_2})$. (See also [4] for recent discussions on the direct construction of the Q-operator of the closed chain.) In [5], a natural set-up to define the Baxter's Q-operator within the quantum inverse scattering method was proposed for the open XXZ chain. However, the generalization of the construction to *off-critical* elliptic solvable models [6] (including the XYZ chain) is still lacking even for the closed chains.

In this paper, we generalize the construction in [5] to Baxter's eight-vertex model and propose that the Q-operator $\bar{Q}(u)$ for the XYZ quantum spin chain (closed or open) is given by the $j\to\infty$ limit of the corresponding transfer matrix $t^{(j)}(u)$ with spin-j (i.e., $(2j+1)$-dimensional) auxiliary space,

$$\bar{Q}(u) = \lim_{j\to\infty} t^{(j)}(u-2j\eta). \tag{1.3}$$

This relation together with the fusion hierarchy of the transfer matrix leads to the T–Q relation. We then use the T–Q relation, together with some additional properties of the transfer matrix, to determine the complete set of the eigenvalues and Bethe Ansatz equations of the transfer matrix associated with the Hamiltonian (1.1) under certain constraint of the boundary parameters (see (5.16) below).

The paper is organized as follows. In Section 2, we introduce our notation and some basic ingredients. In Section 3, we derive the T–Q relation from (1.3) and the fusion hierarchy of the open XXZ chain. In Section 4, some properties of the fundamental transfer matrix are obtained. By means of these properties and the T–Q relation, we in Section 5 determine the eigenvalues of the transfer matrix and the associated Bethe Ansatz equations, thus giving the complete spectrum of the Hamiltonian (1.1) under the constraint (5.16) of the boundary parameters. We summarize our conclusions in Section 6. Some detailed technical calculations are given in Appendices A–C.

Download English Version:

https://daneshyari.com/en/article/1843351

Download Persian Version:

https://daneshyari.com/article/1843351

Daneshyari.com