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Vacuum energies of non-abelian string-configurations in $3+1$ dimensions

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Abstract

We develop a method to compute the fermion contribution to the vacuum polarization energy of string-like configurations in a non-abelian gauge theory. This calculation has been hampered previously by a number of technical obstacles. We use gauge invariance of the energy and separation of length scales in the energy density to overcome these obstacles. We present a proof-of-principle investigation that shows that this energy is small in the $\overline{\text{MS}}$ renormalization scheme. The generalization to other schemes is straightforward.

Keywords: Vacuum polarization energy; Cosmic string; Non-abelian gauge theory; Jost function

1. Introduction

Various field theories suggest the existence of string-like configurations, which are the particle physics analogues of vortices or magnetic flux-tubes in condensed matter physics. Often they are called cosmic strings to distinguish them from the fundamental variables in string theory and to indicate that they stretch over cosmic length scales. If they exist, cosmic strings can potentially have significant cosmological effects. We refer to Ref. [1] for a recent review on the physical implications of strings in the standard model and beyond.

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In the standard model of particle physics, string solutions [2–4] are not topologically stable and thus can only be stabilized dynamically. In exploring the existence of cosmic strings, it is therefore important to be able to accurately calculate their energies. Here we will develop and apply a method to compute the fermion contribution to the leading order quantum correction to the energy, the so-called vacuum polarization or Casimir energy. In a large N_C scenario with many internal degrees of freedom, the fermion contribution dominates that of the bosons.

A number of previous studies have investigated quantum properties of string and vortex configurations, invoking either approximations, simplified configurations, or lower dimensions to cope with technical difficulties. Naculich [5] has shown that in the limit of weak coupling, fermion fluctuations destabilize the non-abelian Z-string. The quantum properties of Z-strings have also been connected to non-perturbative anomalies [6]. The fermionic vacuum polarization energies of QED flux-tubes, and abelian flux-tubes more generally, were investigated using heat-kernel methods [7,8], world line numerics [9] as well as the phase shift method [10]. The heat-kernel method was also used to study self-dual vortices [11]. This method and the world line approach limit renormalization to the subtraction of the divergences in the heat-kernel expansion, thereby obscuring the connection to perturbative renormalization. As we will explain later, the phase shift approach is capable of making straightforward contact with any renormalization condition that is formulated in terms of (momentum space) Green's functions. In lower dimensions the ultra-violet divergences are less severe, which made the computation feasible for the case of two spatial dimensions [12], while the obstacles that arise for the physical case became soon obvious [13]. A first attempt at a full calculation of the quantum corrections to the Z-string energy was carried out in Ref. [14]. Those authors were only able to compare the energies of two string configurations, rather than comparing a single string configuration to the vacuum; these limitations arise from subtleties of the renormalization process that we address in this paper. Also, the contribution of bosonic fluctuations to the vacuum polarization energies has been estimated for vortex configurations using the heat-kernel method [15] and string backgrounds within the phase shift approach [16], which we will use here for the fermionic fluctuations. Stability of cosmic string currents was considered in [17].

We begin by expressing the vacuum polarization energy as the renormalized sum over (half) the change of the single particle energies caused by the localized background. Previously, the fermionic vacuum polarization energy of strings has been computed for the case of two spatial dimensions [12]. The main purpose of the present paper is to extend this computation to the physical case of three spatial dimensions, yielding the vacuum polarization energy per unit length of the string. Though that extension seems straightforward since the string is translationally invariant with respect to this additional coordinate, a number of obstacles arise. They are mainly related to the more complex structure of ultra-violet divergences.

It is well established that the vacuum polarization energies of extended background field configurations, such as solitons or vortices, are unambiguously obtained from a momentum integral that involves the derivative of the phase shifts in the potential generated by the background field [18]. These phase shifts measure the distortion of the spectrum of quantum fluctuations caused by the background. If an object is translationally invariant with respect to a subset of the coordinates, we can use the phase shifts calculated in the non-trivial dimensions combined with appropriate kinematic coefficients to describe the full spectrum of fluctuations [19]. These coefficients vary only with the number of trivial dimensions, but not with the background field. In addition to integrating the result over the magnitude of momentum k, we also need to sum over angular momentum channels. It previously has been shown that for string-type configurations, these two operations are not absolutely convergent, and inappropriately exchanging them may