

Large- N_f chiral transition in the Yukawa model

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Abstract

We investigate the finite-temperature behavior of the Yukawa model in which N_f fermions are coupled with a scalar field ϕ in the limit $N_f \rightarrow \infty$. Close to the chiral transition the model shows a crossover between mean-field behavior (observed for $N_f = \infty$) and Ising behavior (observed for any finite N_f). We show that this crossover is universal and related to that observed in the weakly-coupled ϕ^4 theory. It corresponds to the renormalization-group flow from the unstable Gaussian fixed point to the stable Ising fixed point. This equivalence allows us to use results obtained in field theory and in medium-range spin models to compute Yukawa correlation functions in the crossover regime.

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1. Introduction

The finite-temperature transition in QCD has been extensively studied in the last twenty years and is becoming increasingly important because of the recent experimental progress in the physics of ultrarelativistic heavy-ion collisions. Some general features of the transition, which is associated with the restoration of chiral symmetry, can be studied in dimensionally-reduced three-dimensional models [1,2]. However, a detailed understanding requires a direct analysis in QCD. Being the phenomenon intrinsically nonperturbative, our present knowledge comes mainly from numerical simulations [3,4]. Due to the many technical difficulties—finite-size effects, proper inclusion of fermions, etc.—results are not yet conclusive and thus it is worthwhile to

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study simplified models that show the same basic features but are significantly simpler. In this paper we shall consider a Yukawa model in which N_f fermions are coupled with a scalar field through a Yukawa interaction. The action of the model in $d + 1$ dimensions is

$$S = DN_f \int d^{d+1}\mathbf{x} \left(\frac{1}{2}(\partial\phi)^2 + \frac{\mu}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 \right) + \sum_{f=1}^{N_f} \int d^{d+1}\mathbf{x} \bar{\psi}_f (\not{\partial} + g\phi + M)\psi_f, \quad (1)$$

where $\text{tr} \gamma_\mu^2 = D$ ($D = 2^{d/2}$ if d is even, $D = 2^{(d+1)/2}$ if d is odd), the integration is over $\mathbb{R}^d \times [0, T^{-1}]$, and $\lambda \geq 0$ to ensure the stability of the quartic potential. Along the *thermal* direction we take periodic boundary conditions for the bosonic field ϕ and antiperiodic ones for the fermionic fields ψ_f . The theory must be properly regularized. We shall consider a sharp-cutoff regularization, restricting the momentum integrations in the *spatial* directions to $p < \Lambda$. However, the discussion presented here can be extended without difficulty to any other regularization that maintains at least a remnant of chiral symmetry.

In the limit $N_f \rightarrow \infty$ this model can be solved analytically and one finds that there is a range of parameters in which it shows a transition analogous to that observed in QCD [5,6]. It separates a low-temperature phase in which chiral symmetry is broken from a high-temperature phase in which chiral symmetry is restored. For $N_f = \infty$ this transition shows mean-field behavior, in contrast with general arguments that predict the transition to belong to the Ising universality class. This apparent contradiction was explained in Ref. [7] where, by means of scaling arguments, it was shown that the width of the Ising critical region scales as a power of $1/N_f$, so that only mean-field behavior can be observed in the limit $N_f = \infty$. An analogous behavior was observed in a generalized $O(N)$ σ model in Ref. [8]: for finite values of N the transition was expected to be in the Ising universality class, while the $N = \infty$ solution predicted mean-field behavior. In Ref. [9] we performed a detailed calculation of the $1/N$ corrections, explaining the observed behavior in terms of a critical-region suppression. The analytic technique discussed in Ref. [9] can be applied to model (1). It allows us to obtain an analytic description of the crossover from mean-field to Ising behavior that occurs when N_f is large and to extend the discussion of Ref. [7] to the case $M \neq 0$. More importantly, we are able to show that the phenomenon is universal. In field-theoretical terms, it can be characterized as a crossover between two fixed points: the Gaussian fixed point and the Ising fixed point. This implies that quantitative predictions for model (1) can be obtained in completely different settings. One can use field theory and compute the crossover functions by resumming the perturbative series [10–13]. Alternatively, one can use the fact that the field-theoretical crossover is equivalent to the critical crossover that occurs in models with medium-range interactions [13–17]. This allows one to use the wealth of results available for these spin systems [13–15,17–19]. In this case the interaction range R is essentially equivalent to a power of N , $N \sim R^d$. Finally, we should note that the phenomenon is quite general and occurs in any situation in which there is a crossover from the Gaussian fixed point to a nonclassical stable fixed point. For instance, similar considerations have been recently presented for finite-temperature QCD in some very specific limit [20].

The paper is organized as follows. In Section 2 we review the behavior in the limit $N_f = \infty$. In Section 3 we consider the $1/N_f$ fluctuations and determine the effective theory of the excitations that are responsible for the Ising behavior at the critical point. These modes are described by an effective weakly-coupled ϕ^4 Hamiltonian. In Section 4.1 we present a general discussion of the critical crossover limit. These considerations are applied to the Yukawa model in Sections 4.2 and 4.3. We determine the relevant scaling variables and show how to compute the crossover behavior of the correlation functions. Finally, in Section 5 we present our conclusions.

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