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# Notes on generalised nullvectors in logarithmic CFT

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**Abstract**

In these notes we discuss the procedure how to calculate nullvectors in general indecomposable representations which are encountered in logarithmic conformal field theories. In particular, we do not make use of any of the restrictions which have been imposed in logarithmic nullvector calculations up to now, especially the quasi-primarity of all Jordan cell fields.

For the quite well-studied $c_{p,1}$ models we calculate examples of logarithmic nullvectors which have not been accessible to the older methods and recover the known representation structure. Furthermore, we calculate logarithmic nullvectors in the up to now almost unexplored general augmented $c_{p,q}$ models and use these to find bounds on their possible representation structures.


## 1. Introduction

In recent years the study of conformal field theories (CFTs) which also exhibit indecomposable structures in part of their representations has become an interesting and promising topic of research. A variety of applications of this sensible generalisation of ordinary CFTs, which are also commonly known as logarithmic CFTs, have already surfaced in statistical physics (e.g. [1–4]), in Seiberg–Witten theory (e.g. [5]) and even in string theory (e.g. [6,7]). For a more complete survey of applications pursued so far as well as an introduction to the field see [8–11].

Up to now the main focus of research has been put on a special class of logarithmic CFTs, the $c_{p,1}$ models. The representation theory of their rank 2 indecomposable representations has been analysed completely in [12–15] and a thorough understanding of the representations of the modular group corresponding to the enlarged triplet $\mathcal{W}$-algebra [16] of these models has been

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reached in [11,17,18]. Especially the $c_{2,1} = -2$ model has been understood very well as it is isomorphic to a free field construction of the symplectic fermions [19,20].

But going beyond representation theory we find that the calculation of explicit correlation functions proves to be much more intricate and tedious than in the ordinary CFT case [21,22]. The construction of nullvectors, the key tool in CFT for the calculation of correlation functions, has already been addressed in [23–26] for the case of indecomposable representations. However, the type of these logarithmic nullvectors calculated so far only describes a very special case. Already in the $c_{p,1}$ models the generic logarithmic nullvectors are beyond the scope of this procedure.

On the other hand, the $c_{p,1}$ models are only quite special representatives of the general class of augmented $c_{p,q}$ models [18]. Although these models have already been addressed in some papers (see e.g. [27]), not much is known yet, neither about higher rank representations nor about nullvectors nor about correlation functions. There are, however, good indications that exactly these models might describe important statistical systems, such as perlocation. Especially the augmented $c_{2,3} = 0$ model seems to be of high interest in this respect [4].

The main goal of this paper is to show how to calculate logarithmic nullvectors in general, to give explicit examples and to use the information about the existence of nontrivial logarithmic nullvectors to explore the unknown structure of a more general class of logarithmic CFTs, namely the augmented $c_{p,q}$ models.

In Section 2 we give a short review of the special version of nullvector calculations which has been performed up to now in logarithmic models. In Section 3 we discuss the limitations of this ansatz and propose a more general procedure which is capable of calculating all lowest logarithmic nullvectors in the $c_{p,1}$ models. In particular, our new method does not rely on the quasi-primarity of all Jordan cell fields. In Section 4 we then briefly introduce what we mean by generic augmented $c_{p,q}$ models. We use the methods of the preceding sections with slight modifications in order to obtain constraints on what embedding structures possibly yield rank 2 indecomposable representations in these models. For this we will concentrate on the "smallest" model exhibiting this more generic behaviour, the augmented $c_{2,3} = 0$ model. But any emerging structures should immediately generalise to any $c_{p,q}$ model.

## 2. Jordan cells on lowest weight level

Let us briefly recall the construction of nullvectors in a logarithmic representation in which the states of the Jordan cell are all lowest weight states [8,23,24]. We will especially clarify the respective procedure in [8] and give a proof of the proposed logarithmic nullvector conditions. In the following, we will concentrate on Virasoro representations and try to keep close to the notations of [8].

A Jordan cell of lowest weight states with weight $h$ of rank $r$ is spanned by a basis of states

$$|h; n\rangle = \frac{1}{n!}\theta^n |h\rangle \quad \forall n = 0, \ldots, r-1$$

on which the action of the Virasoro modes is given by

$$L_0|h; n\rangle = h|h; n\rangle + (1 - \delta_{n,0})|h; n-1\rangle \equiv (h + \partial_\theta)|h; n\rangle,$$
$$L_p|h; n\rangle = 0 \quad \forall p > 0.$$

As already defined in [8], $\theta$ is a nilpotent variable with $\theta^r = 0$ and a handy tool to organize the Jordan cell states with the same weight. Due to their almost primary behaviour with the only