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Kink scaling functions in 2D non-integrable quantum field theories

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Abstract

We determine the semiclassical energy levels for the ϕ^4 field theory in the broken symmetry phase on a 2D cylindrical geometry with antiperiodic boundary conditions by quantizing the appropriate finite-volume kink solutions. The analytic form of the kink scaling functions for arbitrary size of the system allows us to describe the flow between the twisted sector of $c = 1$ CFT in the UV region and the massive particles in the IR limit. Kink-creating operators are shown to correspond in the UV limit to disorder fields of the $c = 1$ CFT. The problem of the finite-volume spectrum for generic 2D Landau–Ginzburg models is also discussed.

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1. Introduction

The universal thermodynamical properties of statistical systems with multicritical behavior are described, in mean-field approximation, by appropriate Landau–Ginzburg (LG) field theories:

$$V_l(\phi) = \sum_{k=1}^{l} \lambda_k \phi^{2k-2}, \quad l = 3, 4, \ldots \tag{1.1}$$

Structural (commensurate–incommensurate) phase transitions [1], interface phenomena in ordered and disordered media [2] and phase structure of ferromagnetic systems (see for instance [3]) provide few examples for the applications of the simplest ϕ^4 and ϕ^6 LG models to statistical mechanics and condensed matter physics. In two dimensions, the LG potentials (1.1) appear also in the description of the relevant perturbations of Virasoro minimal models of conformal field theory [4], as well as of the renormalization group flows between them.

The physical quantities associated with a field theory—partition function, energy spectrum, correlation functions, etc.—strongly depend on the geometry of the considered problem (cylindrical, strip, plane, etc.), on the boundary conditions chosen (periodic, Dirichlet, etc.) and on the range of the values of the couplings λ_k. For several integrable quantum field theories in 2D, the above quantities have been exactly computed in finite volume with the so-called Thermodynamics Bethe ansatz method [5] or Destri–deVega equations [6]. These techniques, however, require the integrability of the model, and cannot be applied to the LG theories (1.1), due to their non-integrable nature. In this case, the analysis of the finite-size effects is based on approximate methods as perturbative renormalization group (see [2,3] and references therein), transfer integral techniques [1] and numerical methods.

The low temperature (broken symmetry) phase of these models exhibits, however, specific features—multiple degenerate vacua, non-trivial topological sectors and non-perturbative kink solutions (domain walls)—which require certain improvements of the standard perturbative methods. The non-perturbative semiclassical expansion [7] is known to be an effective method for the quantization of the kink solutions in an infinite volume, independently of the integrability of the model. Its recent extension to finite geometries [8,9] allowed us to derive analytic expressions for the scaling functions of the sine-Gordon model defined on a cylinder with quasi-periodic b.c. (i.e. in the one-kink sector) and on a strip with Dirichlet b.c.'s. It is then natural to address the problem of the finite-size effects in 2D LG models within the context of the semiclassical quantization of kinks in finite volume.

The present paper is devoted to the derivation of the scaling functions of the 2D ϕ^4 theory on a cylindrical geometry with *antiperiodic* b.c. $\phi(x+R) = -\phi(x)$, which for this model corresponds to consider a single kink on the cylinder. This continues our analysis of finite-size effects in the ϕ^4 model, which begun in [10] with the derivation of the finite-volume form factors and spectral functions for the same kind of geometry.

From the mathematical point of view, the derivation of the scaling functions for the ϕ^4 theory on the *twisted* cylinder is analogous to the one performed in [8] for the sine-Gordon model on a cylinder with *quasi-periodic* b.c. This is due to the fact that the finite volume kinks are expressed in both cases in terms of a Jacobi elliptic function, and the computation of the corresponding energy levels is therefore based on the solution of the so-called Lamé equation. Besides a minor technical difference (the equation appears now in a more complicated form, the so-called $N = 2$ Lamé form), an important new feature emerges in the antiperiodic case: the oscillating background cannot be defined for any value of the size of the system, so that the complete description