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ScienceDirect
Nuclear Physics B 813 (2009) 91–105
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

A predictive A_4 model, charged lepton hierarchy and tri-bimaximal sum rule

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Received 20 May 2008; received in revised form 18 December 2008; accepted 29 December 2008

Available online 31 December 2008

Abstract

We propose a novel A_4 model in which the Tri-Bimaximal (TB) neutrino mixing and the charged lepton mass hierarchy are reproduced simultaneously. At leading order, the residual symmetry of the neutrino sector is $Z_2 \times Z_2$ which guarantees the TB mixing without adjusting ad hoc free parameters. In the charged lepton sector, one of the previous Z_2 is maximally broken and the resulting mass matrix is nearly diagonal and hierarchical. A natural mechanism for the required vacuum alignment is given with the help of the supersymmetry and an Abelian symmetry factor. In our model, subleading effects which could lead to appreciable deviations from TB mixing are very restrictive giving rise to possible next-to-leading predictions. From an explicit example, we show that our "constrained" A_4 model is a natural framework, based on symmetry principle, to incorporate the TB sum rule: $\sin^2\theta_{12} = 1/3 + 2\sqrt{2}(\cos\delta \sin\theta_{13})/3$.

1. Introduction

Nowadays continuous improvement on the knowledge of neutrino oscillation parameters makes desirable a neutrino model building going beyond the mere fitting procedure. In particular the leptonic mixing pattern, so different from the one in the quark sector, provides a non-trivial theoretical challenge. The present data [1], at 1σ:

$$\theta_{12} = (34.5 \pm 1.4)^\circ, \qquad \theta_{23} = \left(42.3^{+5.1}_{-3.3}\right)^\circ, \qquad \theta_{13} = \left(0.0^{+7.9}_{-0.0}\right)^\circ, \tag{1}$$

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0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2008.12.025

are fully compatible with the so-called Tri-Bimaximal (TB) mixing matrix:

$$U_{\mathrm{TB}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}, \tag{2}$$

which corresponds to

$$\sin^2\theta_{12} = \frac{1}{3} \quad (\theta_{12} = 35.3^\circ), \qquad \sin^2\theta_{23} = \frac{1}{2}, \qquad \sin^2\theta_{13} = 0. \tag{3}$$

Several interesting ideas leading to a nearly TB mixing have been suggested in the last years [2]. The TB mixing has the advantage of correctly describing the solar mixing angle, which, at present, is the most precisely known. Indeed, its 1σ error, 1.4 degrees corresponds to less than λ_c^2 radians, where $\lambda_c \approx 0.22$ denotes the Cabibbo angle.

TB pattern belongs to the class of mixing textures which are independent on the mass eigenstates. Mass-independent mixing textures usually exhibit an underlying discrete symmetry nature [3]. It has been realized that the TB mixing matrix of Eq. (2) can naturally arise as the result of a particular vacuum alignment of scalars that break spontaneously certain discrete flavour symmetries. A class of very promising models are based on A_4 flavour symmetry [4,5] and subsequently extended to the group T' [6] to cover a reasonable description also for quarks. Despite of the success, the original A_4 models proposed by Altarelli and Feruglio (AF) in [5] require improvement for various reasons. First of all, the leading order results of AF are affected by a large number of subleading corrections. Even though these corrections are hopefully under control, they are totally independent and the model looses the possibility to go beyond the leading prediction. Furthermore the mass eigenvalues are completely unspecified by A_4 and the charged lepton mass hierarchy can be explained only by an extra Froggatt–Nielsen (FN) [7] $U(1)_{\mathrm{FN}}$ factor. Finally, A_4 alone seems unfavorable to accommodate quark masses and eventually be embedded into a GUT theory, despite some recent attempts in this direction: Pati–Salam [8], $SU(5)$ [9,10], $SO(10)$ [11]. Indeed, it is a quite non-trivial task to reproduce all charged fermion hierarchies compatible with a natural mechanism for the vacuum alignment. Particular efforts have been made in this direction. For example, in [12], the problem of fermion hierarchy is partially solved by embedding A_4 into a continuous left–right symmetry. However the question of a natural vacuum alignment in their model remains open. In the recent proposal of A_4 model in a 5D SUSY $SU(5)$ GUT [10], the fermion mass hierarchies and mixing are a result of the interplay of three different sources: a discrete flavour group based on A_4, wave-function suppressions of bulk fields and an additional $U(1)_{\mathrm{FN}}$.

In the literature, there are many mechanisms based on different discrete groups that can successfully describe TB mixing at leading order. Another important issue for the model building is if there were some criteria to distinguish various constructions. One possibility is to go beyond the leading order prediction. There is a remarkable sum rule in the lepton mixing sector first considered in [13,14]:

$$\sin^2\theta_{12} = \sin^2\theta_{12}^{\nu} + \sin^2 2\theta_{12}^{\nu}\cos\delta\sin\theta_{13}, \tag{4}$$

where δ is the Dirac CP violation phase. This sum rule can be derived model-independently when maximal θ_{23} and $\theta_{13} = 0$ come from the neutrino sector at leading order and a non-vanishing θ_{13} arises only when one includes small charged lepton mixing [14,16,17] (under certain assumptions such as θ_{12}^{e} or θ_{13}^{e} dominance [16] as will be explained in Section 5.1). However, it is important to point out that only in flavour models where $\sin^2\theta_{12}^{\nu} = 1/3$ holds almost exactly, this sum rule