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Two-loop electroweak fermionic corrections to $\sin^2\theta_{\text{eff}}^{b\bar{b}}$

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Abstract

We present the first calculation of the two-loop electroweak fermionic correction to the flavour-dependent effective weak-mixing angle for bottom quarks, $\sin^2\theta_{\text{eff}}^{b\bar{b}}$. For the evaluation of the missing two-loop vertex diagrams, two methods are employed, one based on a semi-numerical Bernstein–Tkachov algorithm and the second on asymptotic expansions in the large top-quark mass. A third method based on dispersion relations is used for checking the basic loop integrals. We find that for small Higgs-boson mass values, $M_H \propto 100$ GeV, the correction is sizable, of order $\mathcal{O}(10^{-4})$.

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1. Introduction

Experiments at LEP, SLC and Tevatron have provided a large number of high-precision data, which, being supplemented by detailed studies of higher-order corrections, allow to probe the Standard Model at the loop level and subsequently to predict the mass of the Higgs boson. In this context, the leptonic effective weak-mixing angle, $\sin^2\theta_{\text{eff}}^{\text{lept}}$, plays the most crucial role. It can be defined through the effective vector and axial-vector couplings, g_V^l and g_A^l, of the Z boson to

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leptons (l) at the Z-boson pole,

$$\sin^2\theta_{\rm eff}^{\rm lept} = \frac{1}{4}\left(1 + {\rm Re}\,\frac{g_V^l}{g_A^l}\right). \tag{1}$$

The effective weak-mixing angle can be related to the on-shell Weinberg angle, $\sin^2\theta_w$, as

$$\sin^2\theta_{\rm eff}^{\rm lept} = \sin^2\theta_w\kappa, \tag{2}$$

where $\sin^2\theta_w = 1 - M_W^2/M_Z^2$ and $\kappa = 1 + \Delta\kappa$. At tree level, $\Delta\kappa = 0$ and $\sin^2\theta_{\rm eff}^{\rm lept} = \sin^2\theta_w$. The form factor $\Delta\kappa$ incorporates the higher-order loop corrections. Usually, the W-boson mass, M_W, is not treated as an input parameter but it is calculated from the Fermi constant, G_μ, which is precisely known from the muon lifetime. The relation between M_W and G_μ can be cast in the form

$$M_W^2\left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}(1 + \Delta r), \tag{3}$$

where the quantity Δr [1] contains all higher-order corrections. The presently most accurate calculation of the W-boson mass includes full two-loop and leading higher-order corrections [2]. On the other hand, the quantity κ in Eq. (2) incorporates all corrections to the form factors of the $Zl\bar{l}$ vertex. Recently, the calculation of the two-loop electroweak corrections has been completed [3–7]. The uncertainty on $\sin^2\theta_{\rm eff}^{\rm lept}$ due to unknown higher orders has been estimated to be 0.000047, which is substantially smaller than the error of the current experimental value $\sin^2\theta_{\rm eff}^{\rm lept} = 0.23153 \pm 0.00016$ [8], but still larger than the expected precision, 1.3×10^{-5}, of a future high-luminosity linear collider running at the Z-boson pole [9].

The experimental value for $\sin^2\theta_{\rm eff}^{\rm lept}$ is determined from six asymmetry measurements, $\mathcal{A}_{FB}^{0,l}$, $\mathcal{A}_l(P_\tau)$, $\mathcal{A}_l(\text{SLD})$, $\mathcal{A}_{FB}^{0,b}$, $\mathcal{A}_{FB}^{0,c}$, and $\mathcal{Q}_{FB}^{\rm had}$. Of those, the average leptonic and hadronic measurements differ by 3.2 standard deviations, which is one of the largest discrepancies within the Standard Model. The main impact stems from two measurements, the left–right asymmetry with a polarised electron beam at SLD, $\mathcal{A}_{LR}^0$, and the forward–backward asymmetry for bottom quarks at LEP, $\mathcal{A}_{FB}^{0,b}$. On the experimental side, the only possible source of this discrepancy are uncertainties in external input parameters, in particular parameters describing the production and decay of heavy-flavoured hadrons; see Section 5 of Ref. [8] for a discussion. However, the interpretation of the asymmetry measurements in terms of $\sin^2\theta_{\rm eff}^{\rm lept}$ requires also some theoretical input. The leptonic asymmetries depend on lepton couplings only and can be translated straightforwardly into the leptonic effective weak-mixing angle, with small corrections due to s- and t-channel photon exchange. By contrast, the hadronic observables, $\mathcal{A}_{FB}^{0,c}$, $\mathcal{A}_{FB}^{0,b}$ and $\mathcal{Q}_{FB}^{\rm had}$, depend on the quark couplings, $g_{V,A}^q$. These couplings are associated with a flavour-dependent hadronic effective weak-mixing angle, $\sin^2\theta_{\rm eff}^{q\bar{q}}$,

$$\sin^2\theta_{\rm eff}^{q\bar{q}} = \frac{1}{4|Q_q|}\left(1 + {\rm Re}\,\frac{g_V^q}{g_A^q}\right), \quad q = d, u, s, c, b. \tag{4}$$