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ScienceDirect
Nuclear Physics B 812 [PM] (2009) 402–423
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Exactly solvable D_N-type quantum spin models with long-range interaction

B. Basu-Mallick [a], F. Finkel [b], A. González-López [b,*]

[a] *Theory Group, Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700 064, India*
[b] *Depto. de Física Teórica II, Universidad Complutense, 28040 Madrid, Spain*

Received 24 September 2008; received in revised form 28 October 2008; accepted 4 November 2008

Available online 11 November 2008

Abstract

We derive the spectra of the D_N-type Calogero (rational) su(m) spin model, including the degeneracy factors of all energy levels. By taking the strong coupling limit of this model, in which its spin and dynamical degrees of freedom decouple, we compute the exact partition function of the su(m) Polychronakos–Frahm spin chain of D_N type. In particular, we show that this partition function cannot be obtained as a limiting case of its BC_N counterpart. With the help of the partition function we study several statistical properties of the chain's spectrum, such as the density of energy levels and the distribution of spacings between consecutive levels.

PACS: 75.10.Pq; 75.10.Jm; 05.30.-d; 05.45.Mt

Keywords: Exactly solvable quantum spin models; Partition function; Quantum chaos

1. Introduction

Recent studies of quantum integrable dynamical models and spin chains with long-range interactions [1–8] have not only enriched our understanding of strongly correlated many-particle systems in one dimension, but also influenced several branches of mathematics in a significant way. In particular, it is found that this class of quantum integrable systems have close connections with apparently diverse subjects like generalized exclusion statistics [8–10], quantum Hall effect [11], quantum electric transport in mesoscopic systems [12,13], random matrix theory [14],

* Corresponding author.
E-mail address: artemio@fis.ucm.es (A. González-López).

0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2008.11.006

multivariate orthogonal polynomials [15–17] and Yangian quantum groups [18–20]. The interest in quantum integrable models with long-range interaction was initiated by a seminal work of Calogero [1], where the exact spectrum of an N-particle system on a line with two-body interactions inversely proportional to the square of the distance and subject to a confining harmonic potential was computed in closed form. An exactly solvable trigonometric variant of the rational model introduced by Calogero was proposed shortly afterwards by Sutherland [2,3]. The particles in this so-called Sutherland model move on a circle, with two-body interactions proportional to the inverse square of their chord distances. Subsequently, Olshanetsky and Perelomov established the existence of an underlying A_{N-1} root system structure for both the Calogero and Sutherland models, and constructed generalizations thereof associated with other classical (extended) root systems like B_N, C_N and BC_N [4].

In a parallel development, Haldane and Shastry found an exactly solvable quantum spin-$\frac{1}{2}$ chain with long-range interactions, whose ground state coincides with the $U \to \infty$ limit of Gutzwiller's variational wave function for the Hubbard model, and provides a one-dimensional realization of the resonating valence bond state [5,6]. The lattice sites of this su(2) Haldane–Shastry (HS) spin chain are equally spaced on a circle, all spins interacting with each other through pairwise exchange interactions inversely proportional to the square of their chord distances. A close relation between the HS chain and the su(m) spin generalization of the original (type A) Sutherland model [21–23], which leads to many quantitative predictions, was subsequently established through the so-called "freezing trick" [7,24]. More precisely, it is found that in the strong coupling limit the particles in the spin Sutherland model "freeze" at the coordinates of the equilibrium position of the scalar part of the potential, and the dynamical and spin degrees of freedom decouple. The equilibrium coordinates coincide with the equally spaced lattice points of the HS spin chain, so that the decoupled spin degrees of freedom are governed by the Hamiltonian of the su(m) HS model. Moreover, in this freezing limit the conserved quantities of the spin Sutherland model immediately yield those of the HS spin chain, thereby explaining its complete integrability. By applying the freezing trick to the type A rational Calogero model with spin degrees of freedom, a new integrable spin chain with long-range interaction was constructed in Ref. [7]. The sites of this chain—commonly known in the literature as the Polychronakos or Polychronakos–Frahm (PF) spin chain—are unequally spaced on a line, and in fact coincide with the zeros of the Hermite polynomial of degree N [25]. BC_N versions of both the HS and PF chains were later discussed by several authors [26,27], mainly in connection with their complete integrability.

The powerful technique of the freezing trick was subsequently used to compute the exact partition functions of both the su(m) PF spin chain [28] and the su(m) HS chain [29], the BC_N counterparts of these chains [30,31], and their supersymmetric extensions [32–34]. The exact computation of the partition functions of these quantum integrable spin chains has opened up the exciting possibility of studying various statistical properties of their energy spectra. Indeed, it is found that for a large number of lattice sites the energy level density of such chains follows the Gaussian distribution with a high degree of accuracy [29–31,33–35]. It has also been observed that the distribution of the (normalized) spacings between consecutive energy levels of these chains is not of Poisson type, as might be expected in view of a well-known conjecture of Berry and Tabor [36]. An analytical expression, which explains the unexpected distribution of spacings between consecutive energy levels in the above mentioned chains, has recently been derived using only a few simple properties of their spectra [30].

Our aim in this article is first of all to analyze the spectrum of the su(m) spin Calogero model of D_N type. We shall then apply the freezing trick to compute the exact partition function of the