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Integrable boundary conditions for a non-Abelian anyon chain with $D(D_3)$ symmetry

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Abstract

A general formulation of the Boundary Quantum Inverse Scattering Method is given which is applicable in cases where R-matrix solutions of the Yang–Baxter equation do not have the property of crossing unitarity. Suitably modified forms of the reflection equations are presented which permit the construction of a family of commuting transfer matrices. As an example, we apply the formalism to determine the most general solutions of the reflection equations for a solution of the Yang–Baxter equation with underlying symmetry given by the Drinfeld double $D(D_3)$ of the dihedral group D_3. This R-matrix does not have the crossing unitarity property. In this manner we derive integrable boundary conditions for an open chain model of interacting non-Abelian anyons.
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1. Introduction

The study of systems with non-Abelian anyonic degrees of freedom currently attracts high interest, due to the possibilities for exploiting their topological properties to encode quantum information in a manner which is protected from decoherence [1]. An appropriate framework in which to formulate systems with anyonic symmetries is through the representation theory of quasi-triangular Hopf algebras [2], which includes the class of Drinfeld doubles of finite group algebras [3,4]. This latter class of algebras is particularly suited for the description of non-Abelian anyons where the conjugacy classes and centraliser subgroups of the finite group label generalised notions of the magnetic and electric charges [5,6]. Moreover, within the quasi-

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triangular Hopf algebra framework, consistent braiding and fusion properties for anyonic theories are naturally obtained. The braiding properties are characterised by solutions of the Yang–Baxter equation *without* spectral parameter, which are realised through the universal R-matrix of the algebra. The fusion properties are given by decompositions of tensor product representations of the Hopf algebra, which are governed by the coproduct structure. These fusion rules provide a means to construct interacting systems by assigning energies to the various possible multiplet structures. This then enables the study of one-dimensional (chain) models with local interactions, as has been recently undertaken in [7] using Fibonacci anyons. For this case the interaction energies were chosen in such a way that the local Hamiltonians provided representations of the Temperley–Lieb algebra, which necessarily means that the system is integrable and can be solved exactly. A study of a non-integrable non-Abelian anyon chain can be found in [8].

The theory of integrable chains has a long history associated with the Quantum Inverse Scattering Method (QISM) [9], which relies on a solution of the Yang–Baxter equation *with* spectral parameter to construct a family of commuting transfer matrices. The transfer matrix may be used to generate the conserved operators of an integrable quantum system. Following this procedure we have previously shown that, using the Drinfeld double $D(D_3)$ of the dihedral group D_3, there exists a spectral parameter dependent solution of the Yang–Baxter equation which can be used to construct an integrable interacting non-Abelian anyon chain [10]. There the standard approach of the QISM was used, producing a chain with periodic boundary conditions. For open chain cases integrable boundary conditions are provided by solutions of the reflection equations, as was first elucidated by Sklyanin [11], and is generally known as the Boundary Quantum Inverse Scattering Method (BQISM). Our goal is to extend the BQISM formalism in a manner which will enable the construction of a non-Abelian anyon open chain with integrable boundary conditions.

In Sklyanin's original formulation of the BQISM several conditions were imposed on the R-matrix including P-symmetry, T-symmetry and crossing symmetry [11]. It was soon realised that the BQISM can be extended to cases where the P- and T-symmetry properties are relaxed to the more general PT-symmetry [12], and the crossing symmetry property can be replaced by the more general crossing unitarity condition [13] (see Eq. (10) below for the definition). Later it was shown in [14] that the BQISM can be formulated for cases without PT-symmetry. Here we further extend the formulation of the BQISM by removing the imposition of the crossing unitarity property. This is necessary to construct integrable boundary conditions for the $D(D_3)$ anyon chain, as the R-matrix does not possess this property.

In Section 2 we present the formulation of the BQISM for R-matrices without crossing unitarity. Using the explicit example provided by the $D(D_3)$ R-matrix of [10], in Section 3 we explicitly find the most general solutions of the reflection equations. In Section 4 we use these results to derive a non-Abelian anyon chain with integrable boundary conditions, and concluding remarks are given in Section 5.

2. BQISM for R-matrices without crossing unitarity

Our first objective is to reformulate the BQISM with a minimum number of assumed properties imposed on the R-matrix solution of the Yang–Baxter equation. We start with invertible operators $R(z) \in \mathrm{End}(V \otimes V)$ and $L(z) \in \mathrm{End}(V \otimes W)$ which satisfy the Yang–Baxter equation on $\mathrm{End}(V \otimes V \otimes V)$:

$$R_{12}(xy^{-1})R_{13}(x)R_{23}(y) = R_{23}(y)R_{13}(x)R_{12}(xy^{-1}), \tag{1}$$

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