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Hydrodynamics of $\mathcal{N} = 6$ superconformal Chern–Simons theories at strong coupling

Mohammad R. Garousi [a,b,*], Ahmad Ghodsi [a,b]

[a] *Department of Physics, Ferdowsi University of Mashhad, PO Box 91775-1436, Mashhad, Iran*
[b] *Institute for Research in Fundamental Sciences, PO Box 19395-5531, Tehran, Iran*

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Abstract

Using the duality conjecture between $\mathcal{N} = 6$ supersymmetric $U(N)_{\mathbf{k}} \times U(N)_{-\mathbf{k}}$ Chern–Simons theory and M-theory on $AdS_4 \times S^7/Z_{\mathbf{k}}$, we calculate the corrections to the shear viscosity of the field theory at temperature T. At strong 't Hooft coupling and at small $\mathbf{k}$ level, we have considered one-loop correction to the M-theory effective action. At large $\mathbf{k}$ level, we have considered the α' correction to the type IIA effective action. In both cases the correction to the ratio of shear viscosity to the entropy density is positive.

1. Introduction

One of the most exciting observations from AdS/CFT correspondence is the universality of the ratio of the shear viscosity η to the entropy density s, for any gauge theory with an Einstein gravity dual in the limit of large color N and large 't Hooft coupling λ [1–4]. It has been conjectured in [4] that this universal ratio is the lower bound of all materials at strong couplings, i.e.

$$\frac{\eta}{s} = \frac{1}{4\pi} + \alpha, \tag{1.1}$$

where α is a positive number.

* Corresponding author at: Department of Physics, Ferdowsi University of Mashhad, PO Box 91775-1436, Mashhad, Iran.
E-mail addresses: garousi@mail.ipm.ir (M.R. Garousi), ahmad@mail.ipm.ir (A. Ghodsi).

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The Einstein gravity is not renormalizable, hence, it cannot be considered as a consistent dual theory for a gauge theory. The candidate theory for quantum gravity, i.e. string theory/M-theory, contains Einstein gravity as well as the higher derivative corrections resulting from the stringy or loop effects. These higher derivative terms fix the correction α in the corresponding gauge theory. Using the AdS/CFT duality between $\mathcal{N}=4$ supersymmetric SU(N) Yang–Mills gauge theory and type IIB string theory on $AdS_5\times S^5$, the first stringy effect to the η/s for the $\mathcal{N}=4$ SYM theory has been calculated in [5–7]. The correction is positive, consistent with the conjectured bound.[1]

Recently another example of AdS/CFT correspondence has been proposed by Aharony–Bergman–Jafferies–Maldacena (ABJM) [9]. They consider a particular brane configuration which preserves $\mathcal{N}=3$ supersymmetry. At low energies, by integrating out the massive modes of the brane configuration one finds $U(N)_{\mathbf{k}}\times U(N)_{-\mathbf{k}}$ Chern–Simons conformal field theory which preserves $\mathcal{N}=6$ supersymmetry. This theory is renormalizable and is consistent even at high energies. By lifting the brane configuration to the M-theory they have shown that the gauge theory is equivalent to the low energy theory of N coincident M2-branes in orbifold $R^8/Z_{\mathbf{k}}$. Using the AdS/CFT correspondence, then they have conjectured that the 3-dimensional $\mathcal{N}=6$ superconformal $U(N)_{\mathbf{k}}\times U(N)_{-\mathbf{k}}$ Chern–Simons-matter theory is dual to the M-theory on $AdS_4\times S^7/Z_{\mathbf{k}}$. See [10] for recent studies in different aspects of this duality.

The 't Hooft coupling of this gauge theory is $\lambda=N/\mathbf{k}$. For $\mathbf{k}=1$, theory has no weak coupling regime. The viscosity at large N has been found in [11] to be $\eta=2^{3/2}\pi N^{3/2}T^2/3^3$. For $\mathbf{k}>1$, however, theory has both weak and strong coupling regimes. At strong couplings, the viscosity becomes $\eta=2^{3/2}\pi N^2T^2/(3^3\sqrt{\lambda})$. On the other hand, the entropy of this theory at the supergravity level is [9,12,13] $S=2^{7/2}\pi^2N^2T^2V_2/(3^3\sqrt{\lambda})$ which gives the universal ratio $\eta/s=1/4\pi$. In this paper, we would like to examine the quantum and stringy corrections to this universal value.

The higher derivative corrections to the supergravity in general have field redefinition freedom [17,18], so one may choose different scheme for them. The scheme in which the corrections are written in terms of the 11-dimensional Weyl tensor, modifies the maximal supergravity solution $AdS_4\times S^7$. On the other hand, it has been shown in [19] that the maximal solutions of supergravity are not modified by the higher derivative corrections. Hence, in this scheme the higher derivative corrections associated with the gauge field $F_{(4)}$ influences the solution. In the scheme in which the corrections are written in terms of the 4-dimensional Weyl tensor, the maximal solution is not modified. Hence, it has been argued in [20,21] that this scheme may include all higher derivative corrections associated with the gravity and the gauge field strength $F^{(4)}$.

An outline of this paper is as follows. In Section 2 we briefly review the Minkowski AdS/CFT prescription for calculating the shear viscosity. In Section 3, using the Minkowski AdS/CFT prescription, we calculate the effect of two different schemes of one-loop correction of M-theory effective action to the shear viscosity. In Section 4, using the fact that for large $\mathbf{k}$ level the appropriate description of the gauge theory is the type IIA string theory on $AdS_4\times CP^3$, we calculate two different schemes of α' correction to the shear viscosity. In all cases, the corrections to the classical value of the shear viscosity is positive. Moreover, the corrections to η/s are positive which is consistent with the η/s bound conjecture.

[1] See [8], for a class of four-dimensional gauge theories in which the conjectured lower bound is violated.