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Restoration of supersymmetry on the lattice: Two-dimensional $\mathcal{N} = (2, 2)$ supersymmetric Yang–Mills theory

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Abstract

By numerically investigating the conservation law of the supercurrent, we confirm the restoration of supersymmetry in Sugino's lattice formulation of the two-dimensional $\mathcal{N} = (2, 2)$ supersymmetric $SU(2)$ Yang–Mills theory with a scalar mass term. Subtlety in the case without the scalar mass term, that appears to ruin perturbative power counting, is also pointed out.

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1. Introduction

Nonperturbative study of supersymmetric gauge theory is of great general interest but in the context of lattice formulation, no compelling evidence of supersymmetry has been observed so far. Spacetime lattice is generally irreconcilable with supersymmetry and one must fine-tune coefficients of relevant and marginal operators so that supersymmetric Ward–Takahashi (WT) identities are restored in the continuum limit. (An important exception is the four-dimensional $\mathcal{N} = 1$ supersymmetric Yang–Mills theory (SYM) [1]; see Ref. [2] for a lot of effort went into numerical study of this system; see Ref. [3] for a recent attempt.) Recently, for two- and three-

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dimensional extended supersymmetric gauge theories, lattice formulations that require no (or a little) fine tuning have been proposed [4–18].[1] It can be argued that exact fermionic symmetries in these lattice formulations, combined with other lattice symmetries, (almost) prohibit relevant and marginal supersymmetry breaking operators to appear [5,7]. Supersymmetry is then restored in the continuum limit without (or with a little) fine tuning.[2]

The aim of the present study is to test this scenario of supersymmetry restoration, in Sugino's lattice formulation of the two-dimensional $\mathcal{N}=(2,2)$ $SU(2)$ SYM [7,8]. By a Monte Carlo simulation, we study a supersymmetric WT identity in the form of the supercurrent conservation law. For the reason elucidated in Section 4, we introduce a supersymmetry breaking scalar mass term to the lattice formulation. The expected WT identity hence takes the form of a partially conserved supercurrent ("PCSC") relation in which the breaking term is proportional to the square of the scalar mass. We numerically confirm the restoration of this PCSC relation in the continuum limit. Our result strongly indicates that the proposed scenario for the supersymmetry restoration is in fact valid and the target continuum theory (i.e., the two-dimensional $\mathcal{N}=(2,2)$ $SU(2)$ SYM with a soft supersymmetry breaking mass term) is realized in the continuum limit of the present lattice model.

2. Preliminaries in the continuum theory

The euclidean continuum action of the two-dimensional $\mathcal{N}=(2,2)$ $SU(N_c)$ SYM, that is obtained by dimensionally reducing the $\mathcal{N}=1$ SYM from four to two dimensions, is given by[3]

$$S=\frac{1}{g^2}\int d^2x\ \mathrm{tr}\left\{\frac{1}{2}F_{MN}F_{MN}+\Psi^T C\Gamma_M D_M\Psi+\tilde{H}^2\right\}, \tag{3}$$

where Roman indices M and N run over 0, 1, 2 and 3, while Greek indices μ and ν below run over only 0 and 1. F_{MN} are field strengths $F_{MN}=\partial_M A_N-\partial_N A_M+i[A_M,A_N]$ and covariant derivatives D_M are defined with respect to the adjoint representation, $D_M\Psi=\partial_M\Psi+i[A_M,\Psi]$. Here, in all expressions, it is understood that $\partial_2\to 0$ and $\partial_3\to 0$ (dimensional reduction). We also define complex scalar fields, $\phi\equiv A_2+iA_3$ and $\bar{\phi}\equiv A_2-iA_3$. The auxiliary field $\tilde{H}$ is related to the auxiliary field H in Refs. [7,8] by $\tilde{H}\equiv H-iF_{01}$. We also introduce a soft supersymmetry breaking term

$$S_{\mathrm{mass}}=\frac{1}{g^2}\int d^2x\,\mu^2\,\mathrm{tr}\{\bar{\phi}\phi\}. \tag{4}$$

This term is "soft" in the sense that it does not introduce new ultraviolet divergences compared with the supersymmetric theory S.

[1] For relations among these formulations, see Refs. [19–25].

[2] See also Refs. [26,27] for alternative approaches that do not use exact fermionic symmetries.

[3] In this paper, we adopt the representation

$$\Gamma_0=\begin{pmatrix}-i\sigma_1 & 0\\ 0 & i\sigma_1\end{pmatrix},\quad \Gamma_1=\begin{pmatrix}i\sigma_3 & 0\\ 0 & -i\sigma_3\end{pmatrix},\quad \Gamma_2=\begin{pmatrix}0 & -i\\ -i & 0\end{pmatrix},\quad \Gamma_3=C=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}, \tag{1}$$

and $\Gamma_5\equiv\Gamma_0\Gamma_1\Gamma_2\Gamma_3=\left(\begin{smallmatrix}\sigma_2 & 0\\ 0 & -\sigma_2\end{smallmatrix}\right)$, and set

$$\Psi^T=(\psi_0,\psi_1,\chi,\eta/2) \tag{2}$$

that corresponds to the conventional basis in the topological field theory [28,29]. See also Ref. [30].