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Numerical revision of the universal amplitude ratios for the two-dimensional 4-state Potts model

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Abstract

Monte Carlo (MC) simulations and series expansion (SE) data for the energy, specific heat, magnetization and susceptibility of the ferromagnetic 4-state Potts model on the square lattice are analyzed in a vicinity of the critical point in order to estimate universal combinations of critical amplitudes. The quality of the fits is improved using predictions of the renormalization group (RG) approach and of conformal invariance, and restricting the data within an appropriate temperature window.

The RG predictions on the cancelation of the logarithmic corrections in the universal amplitude ratios are tested. A direct calculation of the effective ratio of the energy amplitudes using duality relations explicitly demonstrates this cancelation of logarithms, thus supporting the predictions of RG.

We emphasize the role of corrections *and* of background terms on the determination of the amplitudes. The ratios of the critical amplitudes of the susceptibilities obtained in our analysis differ significantly from those predicted theoretically and supported by earlier SE and MC analysis. This disagreement might signal that the two-kink approximation used in the analytical estimates is not sufficient to describe with fair accuracy the amplitudes of the 4-state model.

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1. Introduction

In a first paper [1], we studied the universal combinations of critical amplitudes of the 3-state Potts model. The present paper is devoted to a similar analysis in the 4-state case, which is much more involved due to the presence of logarithmic corrections strongly influencing the critical behavior.

We analyze numerical data obtained in Monte Carlo (MC) simulations using the Wolff [2] single-cluster algorithm and also the series expansion (SE) data available in the literature. In the following, we refer to the data type as MC and SE data, respectively. For comparison with our own results, we shall also reconsider the data obtained in MC simulations using the Swendsen–Wang cluster algorithm [3] by Caselle, Tateo, and Vinci [4] and indicated as CTV data.

Our motivation in using different data sets is to achieve a better control of the critical behavior, since one may expect, for the three sources, some differences in the critical region due to the different interplay of the finite size effects. In addition, one can apply different techniques to the data analysis: the fits in the case of the MC data and the approximant technique in the case of the SE data. The consistency of the final results will increase our confidence. We care so much because the presence of logarithmic corrections makes the numerical determination of the critical behavior of the 4-state Potts model a rather delicate task.

To be even safer, two different approaches are used, which were successfully applied also to the $q = 3$ state Potts model in [1]. First we estimate the critical amplitudes, which are then used to compute universal ratios. Second, besides the direct determination of the amplitudes themselves, we estimate *ratios* of critical amplitudes, constructing effective ratio functions, and computing their limiting values at the critical point. This provides a direct estimate of universal ratios. Analyzing the renormalization group equations, we have shown in Appendix A (see also [5,6]) that, in the absence of any regular background term, the logarithmic corrections cancel in the effective ratio functions.

The Hamiltonian of the ferromagnetic Potts model [7] reads as

$$H = -\sum_{\langle ij\rangle} \delta_{s_i s_j}, \tag{1}$$

where s_i is a "spin" variable taking integer values between 0 and $q - 1$, and the sum is restricted to the nearest neighbor sites $\langle ij\rangle$ on a lattice of N sites with periodic boundary conditions. The partition function Z is defined by

$$Z = \sum_{\text{conf}} \mathrm{e}^{-\beta H} \tag{2}$$

with $\beta = 1/k_B T$,[1] and k_B the Boltzmann constant (fixed to unity). On the square lattice in zero magnetic field, the model is self-dual. Denoting by β^* the dual of the inverse temperature β, the duality relation

$$\left(e^{\beta} - 1\right)\left(e^{\beta^*} - 1\right) = q \tag{3}$$

determines the critical value of the inverse temperature [7] $\beta_c = \ln(1 + \sqrt{q}\,) \approx 1.09861$. Dual reduced temperatures τ and τ^* can be defined by

$$\beta = \beta_c(1 - \tau) \quad \text{and} \quad \beta^* = \beta_c(1 + \tau^*). \tag{4}$$

[1] According to the usual terminology, the inverse temperature and the critical exponent of the magnetization are denoted by the same symbol β, since there is no risk of confusion in this context.