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Nuclear Physics B 806 [PM] (2009) 684–714
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

An exactly solvable supersymmetric spin chain of BC_N type

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Received 9 July 2008; accepted 4 August 2008

Available online 19 August 2008

Abstract

We construct a new exactly solvable supersymmetric spin chain related to the BC_N extended root system, which includes as a particular case the BC_N version of the Polychronakos–Frahm spin chain. We also introduce a supersymmetric spin dynamical model of Calogero type which yields the new chain in the large coupling limit. This connection is exploited to derive two different closed-form expressions for the chain's partition function by means of Polychronakos's freezing trick. We establish a boson–fermion duality relation for the new chain's spectrum, which is in fact valid for a large class of (not necessarily integrable) spin chains of BC_N type. The exact expressions for the partition function are also used to study the chain's spectrum as a whole, showing that the level density is normally distributed even for a moderately large number of particles. We also determine a simple analytic approximation to the distribution of normalized spacings between consecutive levels which fits the numerical data with remarkable accuracy. Our results provide further evidence that spin chains of Haldane–Shastry type are exceptional integrable models, in the sense that their spacings distribution is not Poissonian, as posited by the Berry–Tabor conjecture for "generic" quantum integrable systems.

PACS: 75.10.Pq; 05.30.-d; 05.45.Mt

Keywords: Exactly solvable spin chains; Supersymmetry; Quantum chaos

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0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2008.08.014

1. Introduction

In the last few years exactly solvable (or integrable) supersymmetric spin chains and their associated dynamical models have been the subject of extensive research in connection with different topics of current interest, such as the theory of strongly correlated systems [1,2] or the AdS-CFT correspondence [3]. Among these models, the supersymmetric versions of the celebrated Haldane–Shastry chain [4,5] and its rational counterpart proposed by Polychronakos [6] and Frahm [7] occupy a distinguished position, due to the rich mathematical structures at the heart of their highly solvable character. The Haldane–Shastry (HS) chain was introduced in an attempt to construct a simple one-dimensional model whose ground state coincided with Gutzwiller's variational wave function for the Hubbard model in the limit of large on-site interaction [8–10]. It can also be obtained in this limit from the Hubbard model with long-range hopping studied in Ref. [11], in the half-filling regime. The original HS chain describes N spin $1/2$ particles equally spaced on a circle, with pairwise interactions inversely proportional to the square of the chord distance. Its Hamiltonian can be written as

$$\mathcal{H} = J_0 \sum_{i\neq j} \sin(\vartheta_i - \vartheta_j)^{-2} \mathbf{J}_i \cdot \mathbf{J}_j, \quad \vartheta_i \equiv \frac{i\pi}{N}, \tag{1}$$

where $\mathbf{J}_i \equiv \frac{1}{2}(\sigma_i^x, \sigma_i^y, \sigma_i^z)$, σ_i^α is a Pauli matrix at site i, and the summation indices run from 1 to N (as always hereafter, unless otherwise stated). A natural generalization of this chain to su(m) spin [12,13] is obtained by taking $\mathbf{J}_i = (J_i^1, \dots, J_i^{m^2-1})$, where $\{J_i^\alpha\}$ are the generators of the fundamental representation of su(m) at site i. In fact, with the usual normalization $\operatorname{tr}(J_k^\alpha J_k^\gamma) = \frac{1}{2}\delta^{\alpha\gamma}$ we have

$$\mathbf{J}_i \cdot \mathbf{J}_j = \frac{1}{2}\left(S_{ij} - \frac{1}{m}\right),$$

where S_{ij} is the operator permuting the ith and jth spins, so that the su(m) spin Hamiltonian (1) is a linear combination of spin permutation operators.

The spectrum of the spin $1/2$ HS Hamiltonian was numerically analyzed in the original papers of Haldane and Shastry. The more general su(m) spin case was studied in a subsequent publication by Haldane et al. [14], who empirically found a complete description of the spectrum and explained its high degeneracy by an underlying $\mathcal{Y}(\mathrm{sl}_m)$ Yangian symmetry. These results were rigorously established in Ref. [15] by constructing a transfer matrix using the Dunkl operators [16,17] of the (trigonometric) Sutherland dynamical model [18,19]. Although this approach yields an explicit formula for the energies in terms of the so-called *motifs*, the computation of their corresponding degeneracies becomes quite cumbersome when $m > 2$. This is probably the reason why the partition function of this chain was computed only very recently [20], using a procedure known as Polychronakos's *freezing trick* which does not rely on the explicit knowledge of the spectrum [6,21].

The key idea behind the freezing trick is exploiting the connection between the HS chain and the Sutherland spin dynamical model in the strong coupling limit. Indeed, in this limit the particles in the latter model "freeze" at the coordinates of the (unique) equilibrium of the scalar part of the potential, which are exactly the chain sites. Thus, in this limit the dynamical and the spin degrees of freedom decouple, so that the spectrum of the spin Sutherland model becomes approximately the sum of the spectra of the scalar Sutherland model and the HS chain. This observation yields an explicit formula for the partition function of the HS spin chain as the strong