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Charges of dyons in $\mathcal{N}=2$ supersymmetric gauge theory

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Abstract

Expressions for electric and magnetic charges of dyons, which become massless in the strong-coupling limit of the supersymmetric $\mathcal{N}=2$ gauge theory with an arbitrary gauge group are presented. Transitions into different vacua of the $\mathcal{N}=1$ gauge theory, when the $\mathcal{N}=2$ supersymmetry is broken explicitly to the $\mathcal{N}=1$ case, are discussed. The existence of a minimal set of light dyons, which are necessary to describe this transition, is established. The total number of these dyons equals the product of the rank and dual Coxeter number of the gauge group. A conjecture, which states that this minimal set incorporates all possible light dyons, is discussed. A relation of dyon charges with monodromies at weak and strong couplings is outlined and comparison with known charges of dyons for particular gauge groups is made.

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1. Introduction

The properties of dyons, which become massless in the strong-coupling limit in the pure $\mathcal{N}=2$ gauge theory described by the Seiberg–Witten solution are discussed. Explicit simple expressions for magnetic and electric charges of these dyons are written for an arbitrary gauge group. The total number of dyons is shown to depend on two parameters that govern the gauge algebra, its dual Coxeter number and its rank. The number of different massless dyons is shown

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to be related to the Witten index, which equals the number of different vacua in the $\mathcal{N}=1$ supersymmetric gauge theory.

The Seiberg–Witten solution for the $\mathcal{N}=2$ supersymmetric gauge theory [1,2] exploited the idea of S-duality, which expresses physics of strong-coupling phenomena in terms of the weakly coupled light dyons, thus providing an exact description of low-energy properties of the theory for an arbitrary coupling constant. This approach was described with the help of the algebraic curve, which gives the prepotential as an analytical function of the scalar field, as was discussed for the $SU(2)$ gauge group in [1,2]. The idea was extended to cover pure gauge theory with other gauge groups, gauge theory with matter, as well as used to study a number of related new phenomena in the $\mathcal{N}=2$ supersymmetric gauge theory [3–43]. For the classical gauge groups (A, B, C, D series) the curve, which describes the solution is widely believed to be hyperelliptic, though [17] suggested the non-hyperelliptic description for all gauge groups, which is based on the analogy with the integrable systems. Exceptional groups (G, F, E) prove to be more complicated for an analysis, see discussion in [30,33,41].

The prepotential derived from this analysis provides a way to establish the magnetic and electric charges of light dyons, which were presented explicitly for the simplest gauge groups, including $SU(2)$ [1], $SU(3)$ [12], $SU(4)$ and G_2 [36]. Clearly, the charges of light dyons are very interesting by themselves, which inspires their study for a general gauge group. This issue was addressed in [36], which based analysis on [17] that related the Seiberg–Witten solution to the spectral curve of an integrable system. The work [36] provided a general procedure to derive the charges of the dyons, though the expressions found were involving.

The present work considers light dyons using basic properties of the theory directly, avoiding references to the curve that governs the theory. It is known from [1] that light dyons describe the explicit breaking of the $\mathcal{N}=2$ supersymmetry down to $\mathcal{N}=1$ supersymmetry. Therefore matching the known fundamental properties of $\mathcal{N}=2$ and $\mathcal{N}=1$ supersymmetric gauge theories one can extract information related to properties of light dyons. The work is divided into two parts. Sections 2–4 summarize basic properties of the supersymmetric $\mathcal{N}=1,2$ gauge theories. Sections 5–9 derive and discuss expressions for the charges of dyons.

2. Supersymmetric $\mathcal{N}=2$ gauge theories

The supersymmetric $\mathcal{N}=2$ gauge theory includes the scalar field A, two chiral spinors ψ and λ, where the latter represents the gaugino, and the gauge field v_μ, all in the adjoint representation of a gauge group, which is a simple Lie group G with an algebra g [44]. The energy of the scalar field turns zero provided this field has a coordinate independent value that lies in the Cartan subalgebra g_C of the gauge algebra, $A \in \mathsf{g}_\mathrm{C} \subset \mathsf{g}$, and satisfies $\Re(A) \propto \Im(A)$. Thus, the scalar field can develop an expectation value in the vacuum, which makes the vacuum state degenerate, the moduli space is given by g_C, and the scalar field in the vacuum, $A \in \mathsf{g}_\mathrm{C}$, can be treated as an r-dimensional vector, $A \equiv (A_1, \dots, A_r)$.

The vacuum expectation value of the scalar field breaks the gauge symmetry spontaneously. Generically, the symmetry is broken down to r products of gauge $U(1)$, $G \to U(1) \times \cdots \times U(1)$, where r is the rank of the algebra g. There also remains unbroken a discrete group of gauge transformations, which comprises the Weyl group of g, as discussed below. In the perturbation theory region this gauge breaking generates masses for all degrees of freedom, except for those that correspond to the r unbroken $U(1)$ gauge symmetries. As a result, there are r massless gauge fields in the theory, which are similar to photons; each such photon v_μ is accompanied by the corresponding massless fields A, ψ and λ, which all have no electric charges and belong to the