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Resolving disagreement for η/s in a CFT plasma at finite coupling

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Abstract

The ratio of shear viscosity to entropy density in a strongly coupled CFT plasma can be computed using the AdS/CFT correspondence either from equilibrium correlation functions or from the Janik–Peschanski dual of the boost invariant plasma expansion. We point out that the previously found disagreement for η/s at finite 't Hooft coupling is resolved once the incoming-wave boundary condition for metric fluctuations at the horizon of the dual geometry is properly imposed.

1. Introduction

Gauge theory/string theory correspondence of Maldacena [1,2] has been useful in analysis of the transport properties of the strongly coupled gauge theory plasma [3]. In particular, it was proven that the ratio of shear viscosity to the entropy density $\frac{\eta}{s}$ at infinite 't Hooft coupling is universal in all gauge theory plasmas which allow for a holographically dual string theory description [4–7]. At finite 't Hooft coupling (but still in the planar limit), this ratio receives leading contribution from $\mathcal{O}(\alpha'^3)$ string theory corrections to the dual type IIB supergravity background. In [8] it was argued that such corrections are universal as well, as long as the dual gauge theory plasma is conformal.

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The correction to the ratio $\frac{\eta}{s}$ can be computed either from equilibrium correlation functions (as in [9,10]) or by imposing a non-singularity condition of the $\mathcal{O}(\alpha'^3)$ corrected Janik–Peschanski [11] dual of the boost invariant CFT plasma expansion (as in [12]). In the former case it was found that [9,10]

$$\frac{\eta}{s}=\frac{1}{4\pi}\left(1+\frac{135}{8}\zeta(3)\lambda^{-3/2}+\cdots\right), \tag{1.1}$$

while in the latter [12]

$$\frac{\eta}{s}=\frac{1}{4\pi}\left(1+\frac{120}{8}\zeta(3)\lambda^{-3/2}+\cdots\right), \tag{1.2}$$

where λ is the $\mathcal{N}=4$ supersymmetric Yang–Mills 't Hooft coupling.

In this paper we resolve the discrepancy between (1.1) and (1.2). It turns out that the incoming-wave boundary condition on metric fluctuations used to obtain (1.1) were imposed at the supergravity level, rather than at $\mathcal{O}(\alpha'^3)$ string theory corrected background. In what follows we show that once the boundary conditions are properly imposed, the shear viscosity to the entropy ratio obtained from equilibrium correlation functions agrees with (1.2).

2. Incoming wave boundary condition

We consider here the shear quasinormal mode in $\mathcal{O}(\alpha'^3)$ near-extremal D3 brane geometry. Discussion extends to both the sound quasinormal mode and the scalar quasinormal mode.

Equations of motion to the shear quasinormal mode in $\mathcal{O}(\alpha'^3)$ near-extremal D3 brane geometry were derived in [10]. These equations can be expanded perturbatively in $\gamma\equiv\frac{1}{8}\zeta(3)(\alpha')^3$, provided we introduce

$$Z_{\text{shear}}=Z_{\text{shear},0}+\gamma Z_{\text{shear},1}+\mathcal{O}\big(\gamma^2\big). \tag{2.1}$$

We find

$$\begin{aligned}
0&=Z''_{\text{shear},0}+\frac{x^2\mathfrak{q}^2+\mathfrak{w}^2}{x(\mathfrak{w}^2-x^2\mathfrak{q}^2)}Z'_{\text{shear},0}+\frac{\mathfrak{w}^2-x^2\mathfrak{q}^2}{x^2(1-x^2)^{3/2}}Z_{\text{shear},0},\\
0&=Z''_{\text{shear},1}+\frac{x^2\mathfrak{q}^2+\mathfrak{w}^2}{x(\mathfrak{w}^2-x^2\mathfrak{q}^2)}Z'_{\text{shear},1}+\frac{\mathfrak{w}^2-x^2\mathfrak{q}^2}{x^2(1-x^2)^{3/2}}Z_{\text{shear},1}+J_{\text{shear},0}[Z_{\text{shear},0}],
\end{aligned} \tag{2.2}$$

where the source $J_{\text{shear},0}$ is a functional of the zero's order shear mode $Z_{\text{shear},0}$

$$\begin{aligned}
J_{\text{shear},0}[Z_{\text{shear},0}]&=\mathcal{C}^{(4)}_{\text{shear}}\frac{d^4Z_{\text{shear},0}}{dx^4}+\mathcal{C}^{(3)}_{\text{shear}}\frac{d^3Z_{\text{shear},0}}{dx^3}+\mathcal{C}^{(2)}_{\text{shear}}\frac{d^2Z_{\text{shear},0}}{dx^2}\\
&\quad+\mathcal{C}^{(1)}_{\text{shear}}\frac{dZ_{\text{shear},0}}{dx}+\mathcal{C}^{(0)}_{\text{shear}}Z_{\text{shear},0}\\
&\equiv\hat{\mathcal{C}}^{(1)}_{\text{shear}}\frac{dZ_{\text{shear},0}}{dx}+\hat{\mathcal{C}}^{(0)}_{\text{shear}}Z_{\text{shear},0},
\end{aligned} \tag{2.3}$$

where in the second equality we used the first equation in (2.2) to algebraically eliminate the higher derivatives of $Z_{\text{shear},0}$. The coefficients $\mathcal{C}^{(i)}_{\text{shear}}$ are given explicitly in Appendix A of [10]. In (2.2) we introduced

$$\mathfrak{w}=\frac{\omega}{2\pi T_0},\qquad \mathfrak{q}=\frac{q}{2\pi T_0}, \tag{2.4}$$