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High spin limits and non-abelian T-duality

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Abstract

The action of the non-abelian T-dual of the WZW model is related to an appropriate gauged WZW action via a limiting procedure. We extend this type of equivalence to other σ-models with non-abelian isometries and their non-abelian T-duals, focusing on Principal Chiral models. We reinforce and refine this equivalence by arguing that the non-abelian T-duals are the effective backgrounds describing states of an appropriate parent theory corresponding to divergently large highest weight representations. The proof involves carrying out a subtle limiting procedure in the group representations and relating them to appropriate limits in the corresponding backgrounds. We illustrate the general method by providing several non-trivial examples.

Keywords: String theory; T-duality; High spin limits

1. Introduction and conclusions

An important achievement of string theory is that it can describe spacetime physics at the quantum level beyond the General Theory of Relativity. The most appealing class of models admitting an exact string theoretical description is based on coset G/H conformal field theories (CFTs) [1] that admit a spacetime interpretation via the gauged WZW models [2].

In physical applications one deals with field equations. The generic absence, however, of isometries in the gravitational backgrounds corresponding to G/H coset models makes them unsolvable with any of the traditional methods. This deficiency is not a problem in low-dimensional coset models, such as the prototype example of a two-dimensional black hole in [3], or models

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in which the subgroup that is being gauged is abelian. It becomes, nevertheless, a major hurdle when the gauge group is non-abelian (see, for instance, [4,5]).

In recent work we developed a method that overcomes this problem using techniques based on the rich, albeit not manifest, underlying group theoretic structure [6]. We gave the general procedure and, in addition, we presented explicit results for the background corresponding to the $SU(2)_{k_1} \times SU(2)_{k_2}/SU(2)_{k_1+k_2}$ model.

In our present work we focus on the sector of the theory corresponding to representations with divergently large values of highest weight. This is a consistent sector and admits a description in terms of an effective gravitational background, provided that a correlated limit in the levels is taken so that the eigenenergies of the theory remain finite. Based on the specific $SU(2)$ example mentioned above, there are indications [6] that these effective gravitational backgrounds are related to the so-called non-abelian T-duals of the WZW backgrounds. This is further supported by the fact that the gauged WZW action for the coset $(G_k \times H_\ell)/H_{k+\ell}$ is equivalent in the $\ell \to \infty$ limit, to the action for the non-abelian T-dual of the WZW model for G_k with respect to the subgroup H [7].

In the present paper we reinforce this relation by considering the above limit at the level of the states of the theories. Specifically, we construct the eigenstates of the scalar equation for the background fields of the coset theory and carefully take the large spin limit. We demonstrate that these states solve the scalar wave equation for the effective limiting background, or, equivalently, for the non-abelian dual of the original WZW model for G_k. We also extend this equivalence to other σ-models with non-abelian isometries and their non-abelian T-duals focusing, in particular, on Principal Chiral models. In our discussion we present general arguments and give explicit results.

Our results improve our understanding of non-abelian T-duality [8–10] which, unlike the abelian T-duality originating in a string context in [11], has remained in comparison rather poorly understood in spite of a substantial body of work, e.g. [12–20]. In particular, one may now consider these transformations as generating effective backgrounds for describing consistent sectors of some parent theories in the limit of infinite highest weight representations. In fact, this is the physical reason for the fact that the non-abelian T-duality transformation is non-invertible at the level of its path integral formulation.

2. Gauged WZW models and non-abelian T-duality

In this section we briefly review the relation of the gauged WZW models and the non-abelian duals of WZW models at the level of their classical actions.

Consider coset models of the type $(G_k \times H_\ell)/H_{k+\ell}$, with the subgroup H appropriately embedded into the direct product of the groups $G \times H$. The gauged WZW action is [2]

$$\begin{aligned} S_{\mathrm{gWZW}}(g,h,A_\pm) = {} & kI_0(g) + \ell I_0(h) \\ & + \frac{1}{\pi}\int_M \mathrm{Tr}\big[kA_-\partial_+ g g^{-1} + \ell A_-\partial_+ h h^{-1} - kA_+ g^{-1}\partial_- g \\ & - \ell A_+ h^{-1}\partial_- h + kA_- g A_+ g^{-1} + \ell A_- h A_+ h^{-1} - (k+\ell)A_-A_+\big], \end{aligned} \tag{2.1}$$

where g and h are elements of the groups G and H, respectively, parametrized by a total of $\dim(G) + \dim(H)$ variables X^M, and $I_0(g)$ and $I_0(h)$ are the corresponding WZW actions. The