Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics B 799 [FS] (2008) 306–329
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Real-time gauge theory simulations from stochastic quantization with optimized updating

Jürgen Berges [a], Dénes Sexty [a,*]

[a] Institute for Nuclear Physics, Darmstadt University of Technology, Schlossgartenstr. 9, 64289 Darmstadt, Germany

Received 7 August 2007; received in revised form 17 December 2007; accepted 14 January 2008

Available online 30 January 2008

Abstract

We investigate simulations for gauge theories on a Minkowskian space–time lattice. We employ stochastic quantization with optimized updating using stochastic reweighting or gauge fixing, respectively. These procedures do not affect the underlying theory but strongly improve the stability properties of the stochastic dynamics, such that simulations on larger real-time lattices can be performed.

PACS: 11.10.Wx; 04.60.Nc; 05.70.Ln; 11.15.Ha

1. Introduction

First-principles simulations for gauge field theories such as quantum chromodynamics (QCD) on a Minkowskian space–time lattice represent one of the outstanding aims of current research. Typically, calculations are based on a Euclidean formulation, where the time variable is analytically continued to imaginary values. By this the quantum theory is mapped onto a statistical mechanics problem, which can be simulated by importance sampling techniques. In contrast, for real times standard importance sampling is not possible because of a non-positive definite probability measure.

Simulations in Minkowskian space–time, however, may be obtained using stochastic quantization techniques, which are not based on a probability interpretation [1,2]. In Refs. [3,4] this has been recently used to explore the real-time dynamics of an interacting scalar quantum field

* Corresponding author.
E-mail addresses: juergen.berges@physik.tu-darmstadt.de (J. Berges), denes.sexty@physik.tu-darmstadt.de (D. Sexty).

0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2008.01.018

theory and $SU(2)$ gauge field theory in $3+1$ dimensions. In real-time stochastic quantization the quantum ensemble is constructed by a stochastic process in an additional "Langevin-time" using the reformulation for the Minkowskian path integral [5,6]: The quantum fields are defined on a physical space–time lattice, and the updating employs a Langevin equation with a complex driving force in an additional, unphysical "time" direction. Though more or less formal proofs of equivalence of the stochastic approach and the path integral formulation have been given for Minkowskian space–time, not much is known about the general convergence properties and its reliability beyond free-field theory or simple models [6,7]. Most investigations of complex Langevin equations concern simulations in Euclidean space–time with non-real actions [8,9].

In Ref. [4] real-time stochastic quantization was applied to quantum field theory without further optimization. For $SU(2)$ gauge theory no stable physical solution of the complex Langevin equation could be observed even for small couplings. The physical fixed point was found to be approached at intermediate Langevin-times, however, deviations occurred at later times. The onset time for deviations could be delayed and physical results extracted, if the real-time extent of the lattice was chosen to be sufficiently small on the scale of the inverse temperature. This procedure provided severe restrictions for actual applications of the method. In contrast, for self-interacting scalar field theory stable physical solutions were observed.

In this paper we investigate real-time stochastic quantization for gauge theories employing an optimized updating procedure for the Langevin process. We consider optimized updating using stochastic reweighting or gauge fixing, respectively. These procedures do not affect the underlying theory but strongly improve the stability properties of the stochastic dynamics. For $SU(2)$ gauge theory in $3+1$ dimensions we demonstrate that gauge fixing leads already to stable physical solution for not too small $\beta \sim 1/g^2$, where large β correspond to going to the continuum limit of the lattice gauge theory. Where applicable, the results are shown to accurately reproduce alternative calculations in Euclidean space–time. In order to gain analytical understanding and to compare to exact results we also investigate $U(1)$ and $SU(2)$ one-plaquette models.

The paper is organized as follows. In Section 2 we briefly review real-time stochastic quantization for non-Abelian lattice gauge theory following Ref. [4]. The $U(1)$ one-plaquette model of Section 3.1 is used to introduce the concept of stochastic reweighting in Section 3.1.3. The simplicity of the model allows us to compare simulation with analytical results and to investigate in some detail the fixed point structure and convergence properties is Sections 3.1.4 and 3.1.5. In Section 3.2 we consider the $SU(2)$ one-plaquette model and introduce some important notions that will be employed for the optimized updating using gauge fixing for the lattice field theory in Section 4. We present conclusions in Section 5 and an appendix provides some mathematical details.

2. Real-time gauge theory

Gauge theories on a lattice are formulated in terms of the parallel transporter $U_{x,\mu}$ associated with the link from the neighboring lattice point $x+\hat{\mu}$ to the point x in the direction of the lattice axis $\mu = 0, 1, 2, 3$. The link variable $U_{x,\mu} = U^{-1}_{x+\hat{\mu},-\mu}$ is an element of the gauge group G. For $G = SU(N)$ or $U(1)$ one has $U^{-1}_{x,\mu} = U^{\dagger}_{x,\mu}$, however, since we will consider a more general group space in the context of stochastic quantization this will not be assumed. Therefore, we keep $U^{-1}_{x,\mu\nu}$ in the definition of the action, which is described in terms of the gauge invariant plaquette variable

$$U_{x,\mu\nu} \equiv U_{x,\mu} U_{x+\hat{\mu},\nu} U^{-1}_{x+\hat{\nu},\mu} U^{-1}_{x,\nu}, \tag{1}$$