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Gauge symmetry in Kitaev-type spin models and index theorems on odd manifolds

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Abstract

We construct an exactly soluble spin-$\frac{1}{2}$ model on a honeycomb lattice, which is a generalization of Kitaev model. The topological phases of the system are analyzed by study of the ground state sector of this model, the vortex-free states. Basically, there are two phases, A phase and B phase. The behaviors of both A and B phases may be studied by mapping the ground state sector into a general p-wave paired states of spinless fermions with tunable pairing parameters on a square lattice. In this p-wave paired state theory, the A phase is shown to be the strong paired phase, an insulating phase. The B phase may be either gapped or gapless determined by the generalized inversion symmetry is broken or not. The gapped B is the weak pairing phase described by either the Moore–Read Pfaffian state of the spinless fermions or anti-Pfaffian state of holes depending on the sign of the next nearest neighbor hopping amplitude. A phase transition between Pfaffian and anti-Pfaffian states are found in the gapped B phase. Furthermore, we show that there is a hidden SU(2) gauge symmetry in our model. In the gapped B phase, the ground state has a non-trivial topological number, the spectral first Chern number or the chiral central charge, which reflects the chiral anomaly of the edge state. We proved that the topological number is identified to the reduced eta-invariant and this anomaly may be cancelled by a bulk Wess–Zumino term of SO(3) group through an index theorem in $2+1$ dimensions.

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1. Introduction

The concept of the topological order recently is widely interesting the condensed matter physicists because it may describe the different ‘phases’ without breaking any global continuous

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symmetry of the system [1]. However, unlike the conventional order related to the symmetry of the system in Landau's phase transition theory, the topological order of quantum states is not well defined yet. For example, in the quantum Hall effects, the topological property of the quantum stats may be reflected by the filling factor of the Landau level which may be thought as a topological index, the first Chern number in magnetic Brilliouin zone [2,3]. Nevertheless, only the first Chern number cannot fully score the topological order of the quantum Hall states. In a given filling factor, the quasiparticles may obey either Abelian or non-Abelian statistics. On the other hand, the edge state may partially image the topological properties of the bulk state [4]. In quantum Hall system, it was seen that the edge state may be described by a conformal field theory [5]. Thus, according to the bulk-edge correspondence due to the gauge invariance, it shows that the bulk state is determined by a Chern–Simons topological field theory [6]. However, the bridge between the microscopic theory of the two-dimensional electron gas and the Chern–Simons theory was not spanned.

Kitaev recently constructed an exactly soluble spin model in a honeycomb lattice [7]. Using a Majorana fermion representation, he found the quantum state space is characterized by two different topological phases even there is not any global symmetry breaking. The A phase is a gapped phase which has a zero spectral Chern number and the vortex excitations obey Abelian anyonic statistics. The B phase is gapless at special points of Brilliouin zone. When the B phase is gapped by a perturbation, it is topologically non-trivial and has an odd-integer spectral Chern number. (We call the gapless B phase the B1 phase and gapped one the B2 phase.) Kitaev showed that if the spectral Chern number is odd, there must be unpaired Majorana fermions and then the vortex excitations obey non-Abelian statistics. Consistent with the non-Abelian statistics, the fusion rules of the superselection sectors of Kitaev model are the same as those of the Ising model. However, the source of the non-Abelian physics has not been clearly revealed yet. On the other hand, the first Chern number can only relate to an Abelian group and therefore, an odd spectral Chern number leads to a non-Abelian physics but an even one did not is topologically hard to be understood.

Although Kitaev model has a very special spin coupling, its very attractive properties caused a bunch of recent studies [8–20]. It is convenient to understand Kitaev model if one can map this model to a familiar model. In fact, Kitaev model may be mapped into a special p-wave paired BCS state if only the vortex-free sector of the model is considered [11]. We recently generalized Kitaev model to an exactly soluble model whose vortex-free part is equivalent to $\Delta_{1x} p_x + \Delta_{1y} p_y + i(\Delta_{2x} p_x + \Delta_{2y} p_y)$-wave paired fermion states with tunable pairing order parameters Δ_{ab} on a square lattice [12]. The phase diagram of our model has the same shape as that of Kitaev model, i.e., the boundary of the A–B phases are corresponding to the points $\mathbf{p} = (0, 0), (0, \pm\pi)$ and $(\pm\pi, 0)$ in the first Brilliouin zone. The A phase is gapped and may be identified as the strong paring phase of the p-wave paired state [21]. The B phase can be either gapped or gapless even if T-symmetry is broken. We find that gapless excitations in the B phase, i.e., the B1 phase, is protected by a generalized inversion (G-inversion) symmetry under $p_x \leftrightarrow \frac{\Delta_{1y}}{\Delta_{1x}} p_y$ and the emergence of a gapped B(B2) phase is thus tied to G-inversion symmetry breaking. For instance, the $p_x + ip_y$ wave paired state is gapped while $p_y + ip_y$-wave paired state is gapless although they both break the T-symmetry. The critical states of the A–B phase transition remains gapless whether or not T- and G-inversion symmetries are broken, indicative of its topological nature. Indeed, if all Δ_{ab} are tuned to zero, the topological A–B phase transition is from a band insulator to a free Fermi gas. The Fermi surface shrinks to a point zero at criticality.

In this paper, we further generalize the model proposed by the present author and Wang in Ref. [12] to a model whose square lattice mapping includes a next nearest hopping of the spinless