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# Critical phenomena in $\mathcal{N}=4$ SYM plasma

Alex Buchel [a,b,c,*]

[a] *Department of Applied Mathematics, University of Western Ontario, London, ON N6A 5B7, Canada*
[b] *Perimeter Institute for Theoretical Physics, Waterloo, ON N2J 2W9, Canada*
[c] *Albert Einstein Minerva Center, Weizmann Institute of Science, Rehovot 76100, Israel*

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**Abstract**

Strongly coupled $\mathcal{N}=4$ supersymmetric Yang–Mills plasma at finite temperature and chemical potential for an R-symmetry charge undergoes a second order phase transition. We demonstrate that this phase transition is of the mean field theory type. We explicitly show that the model is in the dynamical universality class of 'model B' according to the classification of Hohenberg and Halperin, with dynamical critical exponent $z=4$. We study bulk viscosity in the mass deformed version of this theory in the vicinity of the phase transition. We point out that all available models of bulk viscosity at continuous phase transition are in conflict with our explicit holographic computations.


## 1. Introduction

According to gauge theory/string theory correspondence of Maldacena [1] maximally supersymmetric $\mathcal{N}=4$ $SU(N)$ Yang–Mills (SYM) theory is dual to string theory on $AdS_5 \times S^5$. In the planar limit ($g_{YM}^2 \to 0$, $N \to \infty$ with $\lambda \equiv g_{YM}^2 N$ kept fixed) and for large 't Hooft coupling $\lambda \gg 1$ the strongly coupled SYM is described by classical type IIB supergravity on $AdS_5 \times S^5$, making it essentially soluble. The value of this holographic duality is that it can provide explicit tests of various phenomenological models invented to describe the dynamics of strongly coupled systems. The focus of this paper is the application of gauge/gravity duality to the transport properties of strongly coupled gauge theory plasma in the vicinity of the second order phase transitions.

* Address for correspondence: Department of Applied Mathematics, University of Western Ontario, London, ON N6A 5B7, Canada.
*E-mail address:* abuchel@uwo.ca.

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In [2] it was argued that the only model for the critical behavior of the bulk viscosity in strongly coupled systems at continuous phase transitions not in conflict with explicit holographic computations was that proposed by Onuki [3]. Specifically, Onuki's model predicts that close to the phase transition the bulk viscosity scales as

$$\zeta \propto |t|^{-z\nu+\alpha}, \tag{1.1}$$

where

$$t \equiv \frac{T}{T_c} - 1 \tag{1.2}$$

is the reduced temperature, $\nu$ and $\alpha$ are the usual static critical exponents of the continuous phase transition, and $z$ is a dynamical critical exponent. In this paper we would like to definitely answer the question as to whether or not (1.1) is realized in a strongly coupled gauge theory plasma with a holographic dual.

Our starting point is the best studied example of gauge theory/string theory duality, namely that of $\mathcal{N}=4$ SYM plasma. This theory has an $SO(6) \sim SU(4)$ R-symmetry; thus one can turn on three independent chemical potentials (one for each of the $U(1)$'s in the Cartan subalgebra of the R-symmetry group). It is well known that $\mathcal{N}=4$ SYM plasma at finite temperature $T$ and for a single $U(1)$ R-symmetry[1] chemical potential $\mu$ undergoes a second order phase transition[2] [4–6]. Moreover, recently [7], the conductivity $\sigma_Q$ of this gauge theory plasma was shown to be finite on the critical line

$$\frac{\mu}{T}\bigg|_{critical} = \frac{\pi}{\sqrt{2}}. \tag{1.3}$$

As a result, the authors of [7] argued that the dynamical universality class of $\mathcal{N}=4$ SYM plasma is that of 'model B' according to classification of Hohenberg and Halperin [8], with the dynamical critical exponent

$$z = 4 - \eta, \tag{1.4}$$

with $\eta$ being the anomalous static critical exponent. In this paper we confirm the identification made in [7], and compute $z$ for the $\mathcal{N}=4$ SYM plasma.

Unfortunately, we cannot use the $\mathcal{N}=4$ SYM plasma directly to test Onuki's prediction for the scaling of the bulk viscosity in the vicinity of the phase transition (1.1) — conformal invariance of the theory guarantees that the bulk viscosity must vanish for arbitrary chemical potential and the temperature. Thus, we need to deform the theory in such a way that we break the scale invariance. The simplest deformation one can consider is to give mass $M$ to fermions of $\mathcal{N}=4$ SYM. If

$$M \ll T_{critical}, \tag{1.5}$$

it is sufficient to work to order $\mathcal{O}(\frac{M^2}{T^2})$. Although not necessary, one can think about above deformation (to the order specified) as that corresponding to deforming $\mathcal{N}=4$ plasma to $\mathcal{N}=2^*$ plasma [9–11].

[1] This is not the diagonal $U(1)$ of the $SU(4)$ R-symmetry.

[2] As we show below, some of the static critical exponent first computed in [5] and since then widely used in the literature are incorrect. This issue could be traced back to the fact that the hyperscaling relation between static critical exponents is violated in this theory.