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Statefinder parameters in two dark energy models

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Abstract

The statefinder parameters (r, s) in two dark energy models are studied. In the first, we discuss in four-dimensional General Relativity a two fluid model, in which dark energy and dark matter are allowed to interact with each other. In the second model, we consider the DGP brane model generalized by taking a possible energy exchange between the brane and the bulk into account. We determine the values of the statefinder parameters that correspond to the unique attractor of the system at hand. Furthermore, we produce plots in which we show s, r as functions of red-shift, and the $(s–r)$ plane for each model.

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1. Introduction

A plethora of observational data are now available, which show that we live in a flat universe that expands with an accelerating rate and that the dominant component in the energy budget of the universe is an unusual material, the nature of which still remains unknown. Identifying the origin and nature of dark energy is one of the great challenges in modern theoretical cosmology. The simplest candidate for dark energy is the cosmological constant, which corresponds to a perfect fluid with state parameter $w = p/\rho = -1$. The LCDM model is still in agreement with all observational data. However, due to the problems associated to the cosmological constant, over the years many other theoretical models have been proposed and studied. One class of such

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models is based on some modification of Einstein's gravity [1,2] and one is talking about the so-called geometrical dark energy models. Another class contains the dynamical dark energy models, in which a new dynamical field (almost certainly a scalar field) is coupled to gravity. In the second class one would find models called quintessence [3], phantom [4], quintom [5], k-essence [6], tachyonic [7], etc. A recent review on dark energy dynamics one can find in [8]. Furthermore, an attempt to solve, or at least to alleviate, the coincidence problem is realized in models of interacting dark energy [9]. In this class of models an interaction between the dark matter and dark energy is allowed, which after all is a natural possibility to be considered.

Nowadays several cosmological models of dark energy are available which cannot be excluded by current observational data. However, a few years ago two new cosmological parameters were introduced [10] in order to discriminate between different dark energy models. These are the so-called statefinder parameters and are given entirely in terms of the scale factor and its derivatives with respect to the cosmic time, up to the third order. The statefinder parameters are defined as follows

$$r = \frac{\dddot{a}}{aH^3}, \tag{1}$$

$$s = \frac{r-1}{3(q-\frac{1}{2})} \tag{2}$$

where a is the scale factor of the universe, H is the Hubble parameter, a dot denotes differentiation with respect to the cosmic time t and q is the deceleration parameter

$$q = -\frac{\ddot{a}}{aH^2}. \tag{3}$$

After the deceleration parameter with the second derivative of the scale factor with respect to the cosmic time, a natural quantity to be considered is r with the third derivative of a with respect to t. For the LCDM model, $r = 1$ and $s = 0$. At this point the easiest way to see this is to consider the evolution of the universe at large times, that is when the contribution from matter is negligible. Then the universe expands due to the cosmological constant only and the scale factor grows exponentially with time, $a(t) = \exp(t)$. Then one can immediately see that $r = 1$ and therefore $s = 0$ (since $q = -1$).

The trajectories in the (r–s) plane for various existing models can exhibit quite different behaviors. The deviation of these trajectories from the $(0, 1)$ point defines the distance of a given model from the LCDM model. The statefinder pair (r, s) can successfully differentiate between a wide variety of cosmological models including a cosmological constant, brane models, quintessence, Chaplygin gas, and interacting dark energy models. In a given model the pair r, s can be computed and the trajectory in the (r–s) plane can be drown. Furthermore, the values of r, s can be extracted from future observations [11]. Therefore, the statefinder diagnostic combined with future observations may possibly be used to discriminate between different dark energy models.

Up to now, the statefinder diagnostic has been applied to several models, see e.g. [12]. In the present work we wish to study two dark energy models. In the first we consider in four-dimensional General Relativity a two fluid model with dark matter and dark energy interacting with each other. The form of the interaction is specified below. We do not rely on a concrete particle physics model for dark energy. We just treat dark energy as a hydrodynamical fluid with a constant state parameter $w = p_X/\rho_X$, where p_X, ρ_X are the pressure and energy density of dark energy respectively. In the second model, we consider the DGP brane model generalized by taking into account a possible energy exchange between the brane and the bulk.