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Entanglement as a probe of confinement

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Abstract

We investigate the entanglement entropy in gravity duals of confining large N_c gauge theories using the proposal of [S. Ryu, T. Takayanagi, Phys. Rev. Lett. 96 (2006) 181602, hep-th/0603001; S. Ryu, T. Takayanagi, JHEP 0608 (2006) 045, hep-th/0605073]. Dividing one of the directions of space into a line segment of length l and its complement, the entanglement entropy between the two subspaces is given by the classical action of the minimal bulk hypersurface which approaches the endpoints of the line segment at the boundary. We find that in confining backgrounds there are generally two such surfaces. One consists of two disconnected components localized at the endpoints of the line segment. The other contains a tube connecting the two components. The disconnected surface dominates the entropy for l above a certain critical value l_{crit} while the connected one dominates below that value. The change of behavior at $l = l_{\text{crit}}$ is reminiscent of the finite temperature deconfinement transition: for $l < l_{\text{crit}}$ the entropy scales as N_c^2, while for $l > l_{\text{crit}}$ as N_c^0. We argue that a similar transition should occur in any field theory with a Hagedorn spectrum of non-interacting bound states. The requirement that the entanglement entropy has a phase transition may be useful in constraining gravity duals of confining theories.

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1. Introduction

Consider a $(d+1)$-dimensional quantum field theory (QFT) on $\mathbb{R}^{d+1}$ in its vacuum state $|0\rangle$. Divide the d-dimensional space into two complementary regions,

$$A = \mathbb{R}^{d-1} \times I_l,$$
$$B = \mathbb{R}^{d-1} \times (\mathbb{R} - I_l), \tag{1}$$

where I_l is a line segment of length l. The entanglement entropy between the regions A and B is defined as the entropy seen by an observer in A who does not have access to the degrees of freedom in B, or vice versa (see e.g. [1] for a recent review and references to earlier work). It can be calculated by tracing the density matrix of the vacuum, $\rho_0 = |0\rangle\langle 0|$, over the degrees of freedom in B and forming the reduced density matrix

$$\rho_A = \mathrm{Tr}_B\, \rho_0. \tag{2}$$

The quantum entanglement entropy S_A is then given by

$$S_A = -\,\mathrm{Tr}_A\, \rho_A \ln \rho_A. \tag{3}$$

The above construction can be generalized in a number of ways. In particular, one can replace the vacuum state $|0\rangle$ by any other pure or mixed state, and choose the submanifold of $\mathbb{R}^d$, A, to be different than (1). In this paper we will restrict to the choices above, which are sufficient for our purposes.

The entanglement entropy (3) is in general UV divergent. To leading order in the UV cut-off a it scales like [2,3]

$$S_A \simeq \frac{V_{d-1}}{a^{d-1}} \tag{4}$$

where V_{d-1} is the volume of $\mathbb{R}^{d-1}$ in (1). Note that (4) is independent of l. This turns out to be a general feature—the entropy is defined up to an l independent (infinite) additive constant. In particular, $\partial_l S_A$ and differences of entropies at different values of l approach a finite limit as $a \to 0$. In $(d+1)$-dimensional CFT with $d > 1$, the finite l-dependent part of the entropy is negative and proportional to V_{d-1}/l^{d-1}, while for $d = 1$ it goes like $\ln l$.

If the QFT in question has a gravity dual [4], it is natural to ask whether the entanglement entropy can be calculated using the bulk description. This problem was addressed in [5]. For the case of $(d+1)$-dimensional large N_c conformal field theories with AdS_{d+2} gravity duals, the authors of [5] proposed a simple geometric method for computing the entanglement entropy and subjected it to various tests. This method is to find the minimal area d-dimensional surface γ in AdS_{d+1} such that the boundary of γ coincides with the boundary of A, which in the case (1) consists of two copies of $\mathbb{R}^{d-1}$ a distance l apart. The quantum entanglement entropy between the regions A and B is proportional to the classical area of this surface,

$$S_A = \frac{1}{4G_N^{(d+2)}} \int_\gamma d^d\sigma \sqrt{G_{\mathrm{ind}}^{(d)}}, \tag{5}$$

where $G_N^{(d+2)}$ is the $(d+2)$-dimensional Newton constant and $G_{\mathrm{ind}}^{(d)}$ is the induced string frame metric on γ. Note that the surface γ is defined at a fixed time and (5) gives the entanglement