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Chern–Simons AdS_5 supergravity in a Randall–Sundrum background

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Abstract

Chern–Simons AdS supergravity theories are gauge theories for the super-AdS group. These theories possess a fermionic symmetry which differs from standard supersymmetry. In this paper, we study five-dimensional Chern–Simons AdS supergravity in a Randall–Sundrum scenario with two Minkowski 3-branes. After making modifications to the $D = 5$ Chern–Simons AdS supergravity action and fermionic symmetry transformations, we obtain a $\mathbb{Z}_2$-invariant total action $S = \tilde{S}_{\text{bulk}} + S_{\text{brane}}$ and fermionic transformations $\tilde{\delta}_\epsilon$. While $\tilde{\delta}_\epsilon \tilde{S}_{\text{bulk}} = 0$, the fermionic symmetry is broken by S_{brane}. Our total action reduces to the original Randall–Sundrum model when $\tilde{S}_{\text{bulk}}$ is restricted to its gravitational sector. We solve the Killing spinor equations for a bosonic configuration with vanishing $su(N)$ and $u(1)$ gauge fields.

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1. Introduction

Chern–Simons AdS supergravity [1–3] theories can be constructed only in odd spacetime dimensions. As the name implies, they are gauge theories for supersymmetric extensions of the AdS group.[1] They have a fiber bundle structure and hence are potentially renormalizable [2]. The

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[1] The AdS group in dimension $D \geqslant 2$ is $SO(D-1, 2)$. The corresponding super-AdS groups are given in [3]. For $D = 5$ and $D = 11$, the super-AdS groups are respectively $SU(2, 2|N)$ and $OSp(32|N)$.

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dynamical fields form a single AdS superalgebra-valued connection and hence the supersymmetry algebra closes automatically *off-shell* without requiring auxiliary fields [4]. The Lagrangian in dimension $D = 2n - 1$ is a Chern–Simons $(2n - 1)$-form for the super-AdS connection and is a polynomial of order n in the corresponding curvature. Unlike standard supergravity theories, there can be a mismatch between the number of bosonic and fermionic degrees of freedom.[2] For this reason, the 'supersymmetry' of Chern–Simons AdS supergravity theories is perhaps better referred to as a fermionic symmetry.

$D = 11$, $N = 1$ *Chern–Simons AdS supergravity* may correspond to an off-shell supergravity limit of M-theory [2,3]. It has expected features of M-theory which are not shared by $D = 11$ *Cremmer–Julia–Scherk (CJS) supergravity* [5]. These features include an $osp(32|1)$ superalgebra [6] and higher powers of curvature [7]. *Hořava–Witten theory* [8] is obtained from CJS supergravity by compactifying on an $S^1/\mathbb{Z}_2$ orbifold and requiring gauge and gravitational anomalies to cancel. This theory gives the low energy, strongly coupled limit of the heterotic $E_8 \times E_8$ string theory. In light of the above discussion, it would be interesting to reformulate Hořava–Witten theory with $D = 11$, $N = 1$ Chern–Simons AdS supergravity.

Reformulating Hořava–Witten theory as described above may prove to be difficult. It is simpler to compactify the five-dimensional version of Chern–Simons AdS supergravity on an $S^1/\mathbb{Z}_2$ orbifold and ignore anomaly cancellation issues. Canonical sectors of $D = 5$ Chern–Simons AdS supergravity have been investigated in locally AdS_5 backgrounds possessing a spatial boundary with topology $S^1 \times S^1 \times S^1$ located at infinity [9]. In this paper, as a preamble to reformulating Hořava–Witten theory, we will study $D = 5$ Chern–Simons AdS supergravity in a Randall–Sundrum background with two Minkowski 3-branes [10]. We choose coordinates $x^\mu = (x^{\bar\mu}, x^5)$ to parameterize the five-dimensional spacetime manifold.[3] In terms of these coordinates, the background metric takes the form

$$g_{\mu\nu}\, dx^\mu\, dx^\nu = \mathfrak{a}^2\big(x^5\big)\eta^{(4)}_{\bar\mu\bar\nu}\, dx^{\bar\mu}\, dx^{\bar\nu} + \big(dx^5\big)^2, \tag{1.1}$$

where $\eta^{(4)}_{\bar\mu\bar\nu} = \mathrm{diag}(-1, 1, 1, 1)_{\bar\mu\bar\nu}$, $\mathfrak{a}(x^5) \equiv \exp(-|x^5|/\ell)$ is the *warp factor*, and ℓ is the AdS_5 curvature radius. The coordinate x^5 parameterizes an $S^1/\mathbb{Z}_2$ orbifold, where the circle S^1 has radius ρ and $\mathbb{Z}_2$ acts as $x^5 \to -x^5$. We choose the range $-\pi\rho \leqslant x^5 \leqslant \pi\rho$ with the endpoints identified as $x^5 \simeq x^5 + 2\pi\rho$. The Minkowski 3-branes are located at the $\mathbb{Z}_2$ fixed points $x^5 = 0$ and $x^5 = \pi\rho$. These 3-branes have corresponding tensions $\mathcal{T}^{(0)}$ and $\mathcal{T}^{(\pi\rho)}$ and may support $(3 + 1)$-dimensional field theories.

This paper is organized as follows: In Section 2, we construct a $\mathbb{Z}_2$-invariant bulk theory. This bulk theory is obtained by making modifications to the $D = 5$ Chern–Simons AdS supergravity action and fermionic symmetry transformations which allow consistent orbifold conditions to be imposed. The variation of the resulting bulk action S_{bulk} under the resulting fermionic transformations δ_ϵ vanishes everywhere except at the $\mathbb{Z}_2$ fixed points. We calculate $\delta_\epsilon S_{\text{bulk}}$ in Section 3. In Section 4, we modify S_{bulk} and δ_ϵ to obtain a modified $\mathbb{Z}_2$-invariant bulk theory. The modified bulk action $\tilde{S}_{\text{bulk}}$ is invariant under the modified fermionic transformations $\tilde{\delta}_\epsilon$. In Section 5,

[2] For example, in $D = 5$ *Chern–Simons AdS supergravity* [1], the number of bosonic degrees of freedom $(N^2 + 15)$ is equal to the number of fermionic degrees of freedom $(8N)$ only for $N = 3$ and $N = 5$.

[3] We use indices $\mu, \nu, \ldots = 0, 1, 2, 3, 5$ for local spacetime and $a, b, \ldots = \dot{0}, \dot{1}, \dot{2}, \dot{3}, \dot{5}$ for tangent spacetime. The corresponding metrics, $g_{\mu\nu}$ and $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1, 1)_{ab}$, are related by $g_{\mu\nu} = e_\mu{}^a e_\nu{}^b \eta_{ab}$, where $e_\mu{}^a$ is the fünfbein. Barred indices $\bar\mu, \bar\nu, \ldots = 0, 1, 2, 3$, and $\bar a, \bar b, \ldots = \dot{0}, \dot{1}, \dot{2}, \dot{3}$ denote the four-dimensional counterparts of $\mu, \nu, \ldots$ and $a, b, \ldots$, respectively.