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Spectral analysis of polynomial potentials and its relation with ABJ/M-type theories

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Abstract

We obtain a general class of polynomial potentials for which the Schrödinger operator has a discrete spectrum. This class includes all the scalar potentials in membrane, 5-brane, p-branes, multiple M2 branes, BLG and ABJM theories. We provide a proof of the discreteness of the spectrum of the associated Schrödinger operators. This is the first step in order to analyze BLG and ABJM supersymmetric theories from a non-perturbative point of view.

Keywords: Polynomial potentials; Discrete spectrum; p-branes; Multiple M2 branes; BLG theories; ABJM theories

1. Introduction

There is an intense activity in the spectral characterization of ABJM-type theories at perturbative level. These theories belong to a class of superconformal Chern–Simons gauge theories in three dimensions with $\mathcal{N} = 6$ supersymmetry [1]. The gauge group is $U(N) \times U(N)$ with Chern–Simons level k. The case with gauge group $U(N) \times U(M)$ with different gauge groups $N \neq M$,

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also called ABJ, was considered in [2]. ABJM theories are special cases of the Gaiotto–Witten theories [3], i.e., superconformal Chern–Simons theories with $\mathcal{N} = 4$, in which the supersymmetry is enhanced to $\mathcal{N} = 6$. For the case $N = 2$, the number of supersymmetries is enhanced to $\mathcal{N} = 8$ and it corresponds to the BLG theory [4–6]. In these papers, the fields are evaluated on a 3-algebra with positive inner metric, in terms of a unique finite dimensional gauge group $SO(4)$ with twisted Chern–Simons terms. The ABJM theory or at least a sector of it, can also be recovered from the 3-algebra formulation by relaxing the antisymmetry condition in its structure constants [7].

The interest of these ABJ-like theories is double: on one hand they represent further evidence of the duality AdS_4/CFT_3 [8]. This duality opens an interesting window that allows to compute different aspects of condensed matter in the strong coupling limit in the fields of superconductivity, semiconductors, and so on, unreachable today by other means. For recent reviews on this interesting topic, see [9,10]. On the other hand these calculations also provide results about integrability and finiteness properties of these superconformal Chern–Simons theories in the strong coupling regime. There have been results in this regard recently, see for example [11–16]. For all these reasons, any non-perturbative results related to these theories are of obvious interest.

An important aspect of supermembranes, super 5-branes and supersymmetric multiple M2 branes refers to the quantum stability of the theory and the validity of the Feynman kernel. A natural way to proceed is to formulate the theory on a compact base manifold, perform then a regularization of the theory in terms of an orthonormal basis and analyze properties of the spectrum of the associated Schrödinger operator. This procedure, to start with a field theory and analyze its properties by going to a regularized model, has been very useful in field theory, although relevant symmetries of the theory may be lost in the process. In the case of the $D = 11$ supermembrane [17], an important property of the regularization is that the area preserving diffeomorphism, the residual gauge symmetry of the supermembrane in the light cone gauge, gives rise to a $SU(N)$ gauge symmetry of the regularized model [18,19]. The gauge symmetry of the field theory is then 'represented' as the $SU(N)$ gauge symmetry of the regularized model and it is not lost in the reduction to finite degrees of freedom. The quantum properties of the regularized model is then determined from the Schrödinger operator $-\Delta + V(x) +$ fermionic terms, where the bosonic potential $V(x)$ has the expression

$$V(x) = \sum_i \big[P_i(x)\big]^2. \tag{1}$$

$P_i(x)$ is a homogeneous polynomial on the configuration variables $x \in \mathbb{R}^n$. In the membrane theory $P_i(x)$ are of degree two.

An important aspect of $V(x)$, which determines the spectrum of the associated Schrödinger operator, is the algebraic variety of zero potential which extends to infinity on configuration spaces and the behavior of the potential along that variety. In the case of the (bosonic) membrane the distance between the walls of the valleys along the zero variety goes to zero as we approach infinity, and this was interpreted in [20] as the main reason explaining the discreteness of the spectrum of the membrane Hamiltonian: the wave function cannot escape to infinity. The potential in the transverse directions to the valleys behaves as the potential of an harmonic oscillator. The proof of the discreteness was done in [21] and in [22]. In [21] a bound

$$\langle \Psi, H\Psi \rangle \geqslant \langle \Psi, \lambda\Psi \rangle, \tag{2}$$

in terms of a function $\lambda(x)$, with $\lambda(x) \to \infty$ as $|x| \to \infty$ was obtained. While in [22] a proof of the discreteness of the spectrum of the membrane was done using a theorem of C. Fefferman and