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Regge behavior of gluon scattering amplitudes in $\mathcal{N} = 4$ SYM theory

Stephen G. Naculich [a,*,1], Howard J. Schnitzer [b,2]

[a] *Department of Physics, Bowdoin College, Brunswick, ME 04011, USA*
[b] *Theoretical Physics Group, Martin Fisher School of Physics, Brandeis University, Waltham, MA 02454, USA*

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Abstract

It is shown that the four-gluon scattering amplitude for $\mathcal{N} = 4$ supersymmetric Yang–Mills theory in the planar limit can be written, in both the weak- and strong-coupling limits, as a reggeized amplitude, with a parent trajectory and an infinite number of daughter trajectories. This result is not evident *a priori*, and relies crucially on the fact that the leading IR-divergence and the finite $\log^2(s/t)$-dependent piece of the amplitude are characterized by the same function for all values of the coupling, as conjectured by Bern, Dixon, and Smirnov, and proved by Alday and Maldacena in the strong-coupling limit. We use the Alday–Maldacena result to determine the exact strong-coupling Regge trajectory.

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1. Introduction and conclusion

In this note, we analyze the Regge behavior of the four-gluon scattering amplitude for $\mathcal{N} = 4$ supersymmetric SU(N) Yang–Mills theory in the planar (large-N) limit, using the conjectured ansatz of Bern, Dixon, and Smirnov [1] and the recent strong-coupling results of Alday and Maldacena [2] obtained via the AdS/CFT correspondence. (Other recent applications of this work include Refs. [3,4]. Reggeization of the gluon in nonsupersymmetric Yang–Mills theories [5] as

* Corresponding author.
E-mail addresses: naculich@bowdoin.edu (S.G. Naculich), schnitzr@brandeis.edu (H.J. Schnitzer).
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well as supersymmetric Yang–Mills theories [6] has long been a subject of interest.) The Regge limit corresponds to center-of-mass energy squared $u \to \infty$ with fixed spacelike momentum transfer $s < 0$, where $s = (k_1 + k_2)^2$, $t = (k_1 + k_4)^2$, and $u = (k_1 + k_3)^2$ are Mandelstam variables obeying $s + t + u = 0$. We show that in the Regge limit the color-ordered four-gluon amplitude approaches

$$\mathcal{A}_4 \underset{u\to\infty}{\longrightarrow} \beta(s)\left[\left(\frac{u}{-s}\right)^{\alpha(s)} + \cdots\right] \tag{1.1}$$

where the leading Regge trajectory has the form

$$\alpha(s) = 1 + \frac{1}{4\epsilon} f^{(-1)}(\lambda) - \frac{1}{4} f(\lambda) \log\left(\frac{-s}{\mu^2}\right) + \frac{1}{2} g(\lambda) \tag{1.2}$$

and

$$\beta(s) = (\mathrm{const}) \mathcal{A}^4_{\mathrm{div}}(s) \mathrm{e}^{\tilde{C}(\lambda)} \tag{1.3}$$

with $\cdots$ representing an infinite sum of subleading trajectories. The functions $\alpha(s)$ and $\beta(s)$, like the scattering amplitude itself, exhibit infrared divergences, which we regulate using dimensional regularization in $d = 4 - 2\epsilon$ dimensions. The four-dimensional 't Hooft coupling $\lambda = g^2 N$ is dimensionless, and a scale μ is introduced to allow the coupling to be defined away from four dimensions. The functions $f(\lambda)$ and $g(\lambda)$ characterize the IR divergence of the amplitude [1,2,7]: $f(\lambda)$ is proportional to the cusp anomalous dimension [8], and $g(\lambda)$ is the function $\mathcal{G}_0$ defined in Ref. [1]. The form of $g(\lambda)$ is dependent on the choice of scale μ [2]. Finally $f^{(-1)}(\lambda)$ is defined via

$$\left(\lambda \frac{\mathrm{d}}{\mathrm{d}\lambda}\right) f^{(-1)}(\lambda) = f(\lambda) \tag{1.4}$$

and $\mathcal{A}_{\mathrm{div}}(s)$ and $\tilde{C}(\lambda)$ are defined in Eqs. (2.3) and (2.1).

We emphasize that the Regge behavior of $\mathcal{A}_4$ that we have demonstrated is not *a priori* evident from the results of Refs. [1,2], and in fact appears inconsistent with the fact that the exponent of the amplitude (2.1) goes as $\log^2(t/s)$, whereas Regge behavior would seem to require $\log(t/s)$ dependence. The Regge behavior of the amplitude (1.1) only occurs because the function $f(\lambda)$ that characterizes the leading IR divergence also multiplies the finite $\log^2(s/t)$-dependent piece of the amplitude, as conjectured in Ref. [1].

The Regge trajectory function (1.2) and residue (1.3) are exact (to all orders in the coupling) in the planar limit, depending only on the forms of the functions $f(\lambda)$ and $g(\lambda)$. Since these functions are known in the weak-coupling [1] and strong-coupling [2] limits, we may determine the exact trajectory function explicitly in both these limits. To lowest order in λ, we have

$$\alpha(s) = 1 + \frac{\lambda}{8\pi^2}\left[\frac{1}{\epsilon} - \log\left(\frac{-s}{\mu^2}\right)\right] + \mathcal{O}(\lambda^2). \tag{1.5}$$

This is equivalent to the result found in Ref. [9], where a different regularization scheme was used (see also Refs. [5,6]). The Regge trajectory function in the strong-coupling limit is

$$\alpha(s) \underset{\lambda\to\infty}{\longrightarrow} \sqrt{\lambda}\left[\frac{1}{2\pi\epsilon} - \frac{1}{4\pi}\log\left(\frac{-s}{\mu^2}\right) + \frac{(1-\log 2)}{4\pi}\right] \tag{1.6}$$

where we have used the results of Alday and Maldacena [2].