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Nuclear Physics B 794 (2008) 195–215
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

A dual algorithm for non-Abelian Yang–Mills coupled to dynamical fermions

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Received 7 October 2007; received in revised form 31 October 2007; accepted 1 November 2007

Available online 17 November 2007

Abstract

We extend the dual algorithm recently described for pure, non-Abelian Yang–Mills on the lattice to the case of lattice fermions coupled to Yang–Mills, by constructing an ergodic Metropolis algorithm for dynamic fermions that is local, exact, and built from gauge-invariant boson–fermion coupled configurations. For concreteness, we present in detail the case of three dimensions, for the group $SU(2)$ and staggered fermions, however the algorithm readily generalizes with regard to group and dimension. The treatment of the fermion determinant makes use of a polymer expansion; as with previous proposals making use of the polymer expansion in higher than two dimensions, the critical question for practical applications is whether the presence of negative amplitudes can be managed in the continuum limit.

PACS: 11.15.Ha

Keywords: Latice gauge theory

1. Background

Despite continued progress in algorithms and hardware, the inclusion of dynamical fermions in lattice gauge calculations continues to incur significant computational expense. To motivate our proposal for a novel fermion algorithm, we briefly review how dynamical fermions are currently addressed. Recall that dynamic fermions coupled to a gauge field on a D-dimensional hypercubic lattice for $D \geqslant 2$ are governed by an action of the form

$$S[g_e, \psi_v, \bar{\psi}_v] = S_G[g_e] + S_F[g_e, \psi_v, \bar{\psi}_v], \tag{1}$$

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0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2007.11.006

where the g_e are valued in the gauge group G at the edges of the lattice and ψ_v are the fermion fields defined at the vertices of the lattice.

Unlike gauge group variables, it is not practical to directly simulate Grassmann variables on the computer. A common approach to dynamical fermion simulation starts by integrating out the fermion variables appearing in $S_F[g_e, \psi_v, \bar{\psi}_v]$, to give a function of the gauge variables known as the fermion determinant (its specific form is reviewed in Section 2). The fermion determinant can be combined with the kinetic part of the gauge boson amplitude $e^{-S_G[g_e]}$ to give an effective action for the gauge variables from which simulations on a computer can in principle proceed. However, the fermion determinant renders this effective action non-local—it couples together gauge variables that are arbitrarily distant in the lattice. This poses a considerable problem for the simulation, since computing the change in the effective action due to a small change in any variable becomes very expensive, growing prohibitively with increasing lattice volume. A variety of algorithms have been devised to work with the fermion determinant; a description of some of the methods commonly employed can be found for example in [1].

After reviewing the description of single component, staggered free fermions in terms of self-avoiding polymers (as was done for example in [2]), we review what happens when a similar procedure is applied to multi-component fermion fields minimally coupled to gauge fields. In this case each polymer configuration corresponds to a Wilson loop functional; i.e., the trace of a product of representation matrices around the polymer. Because there is more than one component of the fermion fields in the non-Abelian coupled case, the strict self-avoiding constraint of the single component case is weakened; that is, for an n-component fermion, up to n directed polymer lines can enter and leave a given vertex. The picture has long been known—it is essentially that of a hopping parameter expansion of the fermion determinant, described for example in [3]. Unlike many past applications, in the present case no cut-off in the power of the hopping parameter or otherwise is applied. Because we seek an exactly dual model, all polymers are included in the configurations considered.

For each polymer diagram that arises in the free case, upon applying the duality transformation for the group-valued field the result is a sum of configurations consisting of all closed, branched, colored surfaces (spin foams) with open one-dimensional boundaries defined by the polymer diagram. The totality of spin foams associated with all polymer diagrams (including the trivial empty polymer) defines the joint configuration space. Crucially, local changes to the dual configurations (either polymer or surface structure) lead to local changes in the dual amplitude.

The two theoretical inputs for this construction, a polymer decomposition of the fermion determinant (i.e., hopping parameter expansion as described in [3]) and a dual non-Abelian model (e.g., [4] and references therein), have been present in the literature for some time, and as we shall see the construction of the joint dual model at the formal level is a rather straightforward synthesis of these constituent models. However, unlike (the simplest implementations of) conventional lattice gauge simulations, finding *any* practical algorithm for a dual model has proven somewhat non-trivial in the non-Abelian case for dimensions greater than two. The algorithm proposed here builds upon the dual non-Abelian algorithm of [5] that has recently been tested in the pure Yang–Mills sector. In addition to pure spin foam moves, we construct a set of moves that act on polymer structure and specify the type of vertex amplitudes that arise due to the charges carried by the polymer. Currently, an implementation of this algorithm is being tested and will be reported on in a forthcoming work.

For context, it should be noted that a similar picture was present in the work of Aroca et al. [6] and Fort [7], which dealt with the Abelian case of $U(1)$ and proposed using a Hamiltonian that leads to a different Lagrangian formulation, where the ensemble is built from a restricted