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Harmonicity in $\mathcal{N} = 4$ supersymmetry and its quantum anomaly

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Abstract

The holomorphicity property of $\mathcal{N} = 1$ superpotentials or of $\mathcal{N} = 2$ F-terms involving vector multiplets is generalized to the case of $\mathcal{N} = 4$ 1/2-BPS effective operators defined in harmonic superspace. The resulting *harmonicity* equations are shown to control the moduli dependence of the couplings of higher-dimensional operators involving powers of the $\mathcal{N} = 4$ Weyl superfield, computed by $\mathcal{N} = 4$ topological amplitudes. These equations can also be derived on the string side, exhibiting an anomaly from world-sheet boundary contributions that leads to recursion relations for the *non-analytic* part of the couplings.

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1. Introduction

An important property of $\mathcal{N} = 2$ F-terms involving vector multiplets is holomorphicity, implying that the corresponding couplings are holomorphic functions of the vector moduli. This

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applies, for instance, to the couplings F_g of the higher dimensional F-terms W^{2g}, where W is the self-dual (chiral) Weyl superfield, appearing in the string effective action [1]. On the other hand, the couplings F_g are computed by the topological partition function of the $\mathcal{N}=2$ twisted Calabi–Yau σ-model associated to the six-dimensional compactification manifold of type II string theory in four dimensions [1,2]. It turns out, however, that there is a holomorphic anomaly, related to a violation of the conservation of the BRST current in the topological theory, implying that antichiral fields do not decouple at the quantum level [2–4]. The anomaly arises from boundary contributions and takes the form of an equation that amounts to a recursion relation for the non-holomorphic part of the couplings F_g. From the point of view of the string effective action, it arises from the quantum integration over massless states that is unavoidable when computing on-shell physical amplitudes [1].

These couplings were generalized recently to 1/2-BPS terms of $\mathcal{N}=4$ compactifications, involving powers of the (superdescendant of the) $\mathcal{N}=4$ chiral Weyl superfield $K^{++}=D_-D_-W$, where D_- are particular $SU(4)$ projections of the spinor derivatives. We recall that the $\mathcal{N}=4$ gravity multiplet contains, besides the graviton and the four gravitini, six graviphotons, one complex graviscalar and four spin-1/2 Weyl fermions [5]. Moreover, there is an $SU(4)$ R-symmetry, transforming the gravitini in the fundamental and the graviphotons in the vector representation. The (linearized on-shell) superfield K^{++} satisfies 1/2-BPS shortening conditions. Its lowest component is the (self-dual) graviphoton field strength and its next bosonic components are the (self-dual) Riemann tensor and the second derivative of the graviscalar. Another basic 1/2-BPS superfield in the $\mathcal{N}=4$ theory is the (linearized on-shell) vector multiplet Y^{++}. Its lowest component are the scalar moduli transforming in the vector representation of $SU(4)$, like the graviphoton field strengths.

In [5] two series of 1/2-BPS couplings were found: $\mathcal{F}_g^{(1)}\bar{K}^2K^{2g}$ and $\mathcal{F}_g^{(3)}K^{2(g+1)}$. Here $\mathcal{F}_g^{(1)}$ and $\mathcal{F}_g^{(3)}$ are functions of the $\mathcal{N}=4$ moduli vector multiplets Y^{++} and of the $SU(4)$ harmonic variables that can again be computed by topological amplitudes on $K3\times T^2$ of genus g and $g+1$, respectively. Actually, in six dimensions there is also the series $F_g^{(6d)}W_{6d}^{4g}$, where W_{6d} is a similar Weyl superfield of the six-dimensional gravity multiplet and $F_g^{(6d)}$ is given by a topological theory on $K3$ [6].

In this work, we study the question of what is the analog of $\mathcal{N}=2$ holomorphicity for such 1/2-BPS $\mathcal{N}=4$ couplings. The main novelty of the generalization is that the relevant R-symmetry group becomes non-Abelian, transforming non-trivially the superfields K^{++} and Y^{++}. As a consequence, the notion of chirality of the $\mathcal{N}=2$ theory is replaced by Grassmann analyticity (or 1/2-BPS shortness). The natural framework for studying this problem and covariantizing the expressions is then harmonic superspace [7–9]. By introducing $SU(4)$ harmonic variables one can define K^{++} as a particular harmonic projection of the sixplet of superfields $K_{ij}=-K_{ji}=D_iD_jW$, associated to a corresponding 1/2-BPS subspace of the full $\mathcal{N}=4$ superspace. Supersymmetry then implies that the coupling coefficients $\mathcal{F}_g$ are functions of the same harmonic projected vector superfields Y^{++} living in the same 1/2-BPS subspace. Thus, $\mathcal{F}_g(Y^{++})$ is independent of the five remaining projections of the sixplet of the scalar moduli. This defines a notion of analyticity that naturally generalizes $\mathcal{N}=2$ holomorphicity for the chiral $\mathcal{N}=2$ vector multiplets.

In this work, we show that the above property of analyticity can be formulated in terms of a set of differential constraints on the couplings $\mathcal{F}_g$ of $\mathcal{N}=4$ 1/2-BPS effective operators. They express the property of the analytic functions $\mathcal{F}_g(Y^{++})$ that, when expanded in powers of the harmonic variables and the scalar fields, the coefficients should form symmetric trace-