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Four-fermion production near the W pair-production threshold

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Abstract

We perform a dedicated study of the four-fermion production process $e^-e^+ \to \mu^-\bar{\nu}_\mu u\bar{d}X$ near the W pair-production threshold in view of the importance of this process for a precise measurement of the W boson mass. Accurate theoretical predictions for this process require a systematic treatment of finite-width effects. We use unstable-particle effective field theory (EFT) to perform an expansion in the coupling constants, Γ_W/M_W, and the non-relativistic velocity v of the W boson up to next-to-leading order in $\Gamma_W/M_W \sim \alpha_{\mathrm{ew}} \sim v^2$. We find that the dominant theoretical uncertainty in M_W is currently due to an incomplete treatment of initial-state radiation. The remaining uncertainty of the NLO EFT calculation translates into $\delta M_W \approx 10$–15 MeV, and to about 5 MeV with additional input from the NLO four-fermion calculation in the full theory.

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1. Introduction

The mass of the W gauge boson is a key observable in the search for virtual-particle effects through electroweak precision measurements. Its current value, $\hat{M}_W = (80.403 \pm 0.029)$ GeV [1], is determined from a combination of continuum W pair-production at LEPII and single-W

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production at the Tevatron.[1] Further measurements of single-W production at the LHC should reduce the error by a factor of two. Beyond LHC it has been estimated that an error of 6 MeV could be achieved by operating an e^-e^+ collider in the vicinity of the W pair-production threshold [3]. This estimate is based on statistics and the performance of a future linear collider, and it assumes that the cross section is known theoretically to sufficient accuracy so that its measurement can be converted into one of M_W. In reality, achieving this accuracy is a difficult theoretical task, requiring the calculation of loop and radiative corrections. Since the W bosons decay rapidly, this calculation should be done for a final state of sufficiently long-lived particles, rather than for on-shell W pair-production. A systematic treatment of finite-width effects is therefore needed.

In this paper we investigate in detail the inclusive four-fermion production process

$$e^-(p_1)e^+(p_2) \to \mu^-\bar{\nu}_\mu u\bar{d} + X \tag{1}$$

in the vicinity of the W pair-production threshold, i.e., for $s \equiv (p_1 + p_2)^2 \sim 4M_W^2$. Here X denotes an arbitrary flavour-singlet state (nothing, photons, gluons, ...). No kinematic cuts shall be applied to the final state. In this kinematical regime the process (1) is primarily mediated by the production of two resonant, non-relativistic W bosons with virtuality of order

$$k^2 - M_W^2 \sim M_W^2 v^2 \sim M_W \Gamma_W \ll M_W^2, \tag{2}$$

one of which decays into leptons, the other into hadrons. Here we have introduced the non-relativistic velocity v, and the W decay width Γ_W. We perform a systematic expansion of the total cross section in the small quantities

$$\alpha_{\mathrm{ew}}, \qquad \frac{s - 4M_W^2}{4M_W^2} \sim v^2, \qquad \frac{\Gamma_W}{M_W} \sim \alpha_{\mathrm{ew}}, \tag{3}$$

corresponding to a (re-organized) loop expansion and a kinematic expansion. All three expansion parameters are of the same order, and for power-counting purposes we denote them collectively as δ. Our calculation is accurate at next-to-leading order (NLO). Note that resonant processes such as (1) are complicated by the need to account for the width of the intermediate unstable particles to avoid kinematic singularities in their propagators. The expansion in the electroweak coupling $\alpha_{\mathrm{ew}} = \alpha/s_w^2$ is therefore not a standard loop expansion. (α denotes the electromagnetic coupling, and $s_w^2 \equiv \sin^2\theta_w$ with θ_w the Weinberg angle.)

NLO calculations of four-fermion production have been done already some time ago in the continuum (not near threshold) in the double-pole approximation for the two W propagators [4–6] or with further simplifications [7,8]. This approximation was supposed to break down for kinematic reasons in the threshold region. Thus, when this project was begun [9], there existed only LO calculations in the threshold region as well as studies of the effect of Coulomb photon exchanges [10,11], rendering the effective field theory approach [12–14] the method of choice for the NLO calculation. Meanwhile a full NLO calculation of four-fermion production has been performed in the complex mass scheme [15,16] without any kinematic approximations, and for the fully differential cross sections in the continuum or near threshold. This is a difficult calculation that required new methods for the numerical evaluation of one-loop six-point tensor integrals. In comparison, our approach is computationally simple, resulting in an almost analytic

[1] This value refers to the definition of the W mass from a Breit–Wigner parameterization with a running width as it is adopted in the experimental analyses. It is related to the pole mass M_W used in this paper by [2] $\hat{M}_W - M_W = \Gamma_W^2/(2M_W) + O(\alpha_{\mathrm{ew}}^3)$.