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Nuclear Physics B 825 [FS] (2010) 366–395

NUCLEAR PHYSICS B

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The coupling of Chern–Simons theory to topological gravity

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Received 26 May 2009; accepted 18 September 2009

Available online 22 September 2009

Abstract

We couple Chern–Simons gauge theory to 3-dimensional topological gravity with the aim of investigating its quantum topological invariance. We derive the relevant BRST rules and Batalin–Vilkovisky action. Standard BRST transformations of the gauge field are modified by terms involving both its anti-field and the super-ghost of topological gravity. Beyond the obvious couplings to the metric and the gravitino, the BV action includes hitherto neglected couplings to the super-ghost. We use this result to determine the topological anomalies of certain higher ghost deformations of $SU(N)$ Chern–Simons theory, introduced years ago by Witten. In the context of topological strings these anomalies, which generalize the familiar framing anomaly, are expected to be cancelled by couplings of the closed string sector. We show that such couplings are obtained by dressing the closed string field with topological gravity observables.

PACS: 11.15.-q; 11.25.Tq; 11.25.Uv

Keywords: Chern–Simons gauge theory; Topological gravity; Topological anomalies; Batalin–Vilkovisky quantization

1. Introduction and summary

Classical Chern–Simons (CS) theory [1] on a 3-dimensional manifold M_3 is both gauge invariant and invariant under space–time diffeomorphisms, without the need to introduce a space–time metric $g_{\mu\nu}$. In the quantum theory the metric appears in the gauge-fixing term: The issue regarding topological anomalies is whether or not quantum averages depend on the chosen metric.

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0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2009.09.022

It was understood by Witten, in his celebrated paper [2], that quantum CS theory indeed suffers from a topological anomaly, the so-called *framing* anomaly.[1] In the present work we address the question of quantum topological invariance in certain generalizations of CS gauge theory which were introduced by Witten in [4] and which are obtained by adding observables with higher ghost number to the classical CS action.

We will formulate the problem by making use of a trick which goes back to the early days of BRST renormalization methods[2] and which can be explained as follows. Consider a gauge-fixed action

$$\Gamma\left(\alpha^i\right) = \Gamma_0 + S_0 \chi\left(\alpha^i\right) \tag{1.1}$$

where Γ_0 is the classical action, S_0 is the BRST operator, $\chi(\alpha^i)$ is the gauge-fermion which depends on the commuting parameters α^i. To simplify the notation we dropped any references to either fields or sources. The goal is understanding the (in)dependence on α^i of quantum averages, which we schematically denote as

$$Z\left(\alpha^i\right) = \int \mathrm{e}^{\frac{i}{\hbar}\Gamma(\alpha^i)} \tag{1.2}$$

To study this question one extends the action of S_0 to the parameters α^i by defining a new nilpotent BRST operator $\hat{S}$

$$\hat{S} \equiv S_0 + \beta^i \partial_{\alpha^i} \tag{1.3}$$

where β^i are anti-commuting variables. The $\hat{S}$-invariant classical action

$$\hat{\Gamma}\left(\alpha^i, \beta^i\right) \equiv \Gamma_0 + \hat{S}\chi\left(\alpha^i\right) \tag{1.4}$$

defines a new "partition function" which, in general, depends on both α^i and β^i:

$$Z\left(\alpha^i, \beta^i\right) = \int \mathrm{e}^{\frac{i}{\hbar}\hat{\Gamma}(\alpha^i,\beta^i)} \tag{1.5}$$

$Z(\alpha^i, \beta^i)$ satisfies, *up to anomalies*, the identity

$$\hat{S} Z\left(\alpha^i, \beta^i\right) = 0 \tag{1.6}$$

It admits an expansion in terms of the anti-commuting variables

$$Z\left(\alpha^i, \beta^i\right) = Z^{(0)}\left(\alpha^i\right) + \beta^i Z^{(1)}_i\left(\alpha^i\right) + \beta^i \beta^j Z^{(2)}_{ij}\left(\alpha^i\right) + \cdots \tag{1.7}$$

whose first term is the original quantum average one is interested in

$$Z^{(0)}\left(\alpha^i\right) = Z\left(\alpha^i\right) \tag{1.8}$$

The identity (1.6) translates into identities for each one of the $Z^{(k)}_{i_1 \ldots i_k}(\alpha^i)$. The first one, with $k = 0$, is:

$$\beta^i \frac{\partial}{\partial \alpha_i} Z^{(0)}\left(\alpha^i\right) = 0 \;\Rightarrow\; \frac{\partial}{\partial \alpha_i} Z\left(\alpha^i\right) = 0 \tag{1.9}$$

[1] For a discussion which expands on the local presentation of the framing anomaly, the point of view of the present paper, see also [3].

[2] This idea, which appears to have been known to BRST experts for quite a long time, was rediscovered several times in different contexts. It was applied, for example, in [5] to Yang–Mills theory and in [6] to supersymmetry.