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Nuclear Physics B 825 [FS] (2010) 396–425

NUCLEAR PHYSICS B

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Exact form factors of the $SU(N)$ Gross–Neveu model and $1/N$ expansion

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Received 7 July 2009; accepted 18 September 2009

Dedicated to the memory of Alexey Zamolodchikov

Abstract

The general $SU(N)$ form factor formula is constructed. Exact form factors for the field, the energy–momentum and the current operators are derived and compared with the $1/N$-expansion of the chiral Gross–Neveu model and full agreement is found. As an application of the form factor approach the equal time commutation rules of arbitrary local fields are derived and in general anyonic behavior is found.

PACS: 11.10.-z; 11.10.Kk; 11.55.Ds

Keywords: Integrable quantum field theory; Form factors

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1. Introduction

Quantum chromodynamics, the theory of the strong interactions, is a non-Abelian gauge theory based on the gauge group $SU(3)$. It was first pointed out by 't Hooft [1,2] that many features of QCD can be understood by studying a gauge theory based on the gauge group $SU(N)$ in the limit $N \to \infty$. One might think that letting $N \to \infty$ would make the analysis more complicated because of the larger gauge group and consequently increase in the number of dynamical degrees

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doi:10.1016/j.nuclphysb.2009.09.023

of freedom. Also one can think that $SU(N)$ gauge theory has very little to do with QCD because $N \to \infty$ is not close to $N = 3$. However it is well known that the $1/N$ expansion provides good results which can be compared with experiments [3].

One of the most important trends in theoretical physics in the last decades is the development of exact methods which are completely different from perturbation theory. Resolution of the strong coupling problem would give us a full understanding of the structure of interactions in non-Abelian gauge theory. One promising possibility of overcoming the limitations of perturbation theory is the application of exact integrability. From this point of view the two-dimensional integrable quantum field theories are in a sense a laboratory for investigations of those properties of quantum field theories, which cannot be described via perturbation theory.

The chiral $SU(N)$ Gross–Neveu [4] model given by the Lagrangian

$$\mathcal{L} = \sum_{i=1}^{N} \bar{\psi}_i i\gamma\partial\psi_i + \frac{g^2}{2}\left(\left(\sum_{i=1}^{N} \bar{\psi}_i \psi_i\right)^2 - \left(\sum_{i=1}^{N} \bar{\psi}_i \gamma^5 \psi_i\right)^2\right) \tag{1}$$

is an interesting $(1+1)$-dimensional field theory that can be studied using the $1/N$ expansion. The model is asymptotically free with a spontaneously broken chiral symmetry, and so shares some dynamical features with QCD. Gross and Neveu [4] investigated the model using an $1/N$ expansion. Apparently a chiral $U(1)$-symmetry is spontaneously broken, the fermions acquire mass and a Goldstone boson seems to appear. This is of course not possible in two space–time dimensions and severe infrared divergences appear due the "would-be-Goldstone boson". However, it has been argued by Witten [5] that dynamical mass generation can be reconciled with the absence of spontaneous symmetry breaking. There exist further (different) approaches to overcome these problems and to formulate a $1/N$ expansion [6–8] (see also [9]). On shell they all agree and are consistent with the exact S-matrix (2). We follow here the approach of Swieca et al. [8] where additional fields are introduced in order to compensate the infrared divergences. The authors claim that since the physical fermions have lost not only the chiral $U(1)$ but also the charge $U(1)$ symmetry, they transform accordingly to pure $SU(N)$. They propose an interpretation of the antiparticles as a bound state of $N-1$ particles. Furthermore this means that the particles satisfy neither Fermi nor Bose statistics, but rather carry "spin" $s = \frac{1}{2}(1 - 1/N)$. As a consequence there are unusual crossing relations and Klein factors.

In this article we will focus on the $SU(N)$ Gross–Neveu form factors using the "bootstrap program" [10,11]. We provide here some examples, calculate the form factors exactly and compare the results with field theoretical $1/N$ expansions. We emphasize that in addition to the operators in the vacuum sector, such as the energy–momentum tensor and the current, we also consider anyonic operators as the fundamental fields. We also derive the equal time commutation rules for local operators which are in particular complicated due to the unusual crossing formulae related to the Klein factors.

The general form factor of an operator $\mathcal{O}(x)$ for n-particles, which is a co-vector valued function and can be written as [12]

$$F_{\underline{\alpha}}^{\mathcal{O}}(\underline{\theta}) = K_{\underline{\alpha}}^{\mathcal{O}}(\underline{\theta}) \prod_{1 \leqslant i < j \leqslant n} F(\theta_{ij})$$

where $\underline{\theta} = (\theta_1, \ldots, \theta_n)$ is the set of rapidities of the particles $\underline{\alpha} = (\alpha_1, \ldots, \alpha_n)$. The scalar function $F(\theta)$ is the minimal form factor function and the K-function $K_{\underline{\alpha}}^{\mathcal{O}}(\underline{\theta})$ contains the entire pole structure and its symmetry is determined by the form factor equations (i) to (v) [13]. To construct the K-function we must apply the nested off-shell Bethe ansatz to capture the vectorial