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Universal properties of frustrated spin systems: $1/N$-expansion and renormalization group approaches

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Abstract

We consider a quantum two-dimensional $O(N)\otimes O(2)/O(N-2)\otimes O(2)_{\text{diag}}$ nonlinear sigma model for frustrated spin systems and formulate its $1/N$-expansion which involves fluctuating scalar and vector fields describing kinematic and dynamic interactions, respectively. The ground state phase diagram of this model is obtained within the $1/N$-expansion and $2+\varepsilon$ renormalization group approaches. The temperature dependence of correlation length in the renormalized classical and quantum critical regimes is discussed. In the region $\rho_{\text{in}}<\rho_{\text{out}}$, $\chi_{\text{in}}<\chi_{\text{out}}$ of the symmetry broken ground state ($\rho_{\text{in,out}}$ and $\chi_{\text{in,out}}$ are the in- and out-of-plane spin stiffnesses and susceptibilities) the mass M_μ of the vector field can be arbitrarily small, and physical properties at finite temperatures are universal functions of $\rho_{\text{in,out}}$, $\chi_{\text{in,out}}$, and temperature T. For small enough M_μ these properties show a crossover from low- to high temperature regime at $T\sim M_\mu$. In the region $\rho_{\text{in}}>\rho_{\text{out}}$ or $\chi_{\text{in}}>\chi_{\text{out}}$ finite-temperature properties are universal functions only at sufficiently large M_μ. The high-energy behaviour in the latter region is similar to the Landau-pole dependence of the physical charge e on the momentum scale in quantum electrodynamics, with mass M_μ playing a role of e^{-1}. The application of the results obtained to the triangular-lattice Heisenberg antiferromagnet is considered.

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1. Introduction

The description of frustrated systems is a long-discussed subject [1–4]. These systems pose an important and interesting problem of condensed matter physics, and also provide a test of various methods of quantum field theory [5–8]. Competing interactions leading to frustration favor strong quantum fluctuations and therefore tune the system towards quantum phase transition (QPT) into the phase without spontaneous symmetry breaking.

Examples of frustrated systems are the Heisenberg triangular lattice antiferromagnets (TLAF). In particular, VCl_2 [9], VBr_2 [10,11], and recently investigated Li_7RuO_6 [12] are layered compounds with the triangular lattice structure within a layer. The corresponding Heisenberg model with nearest-neighbour exchange interaction is shown to have a non-collinear ordered ground state with the sublattice magnetization suppressed by quantum fluctuations [13–16]. TLAF can be further tuned to QPT into the spin-liquid state by including additional next-nearest [17] or ring exchange interactions [18], impurities or charge fluctuations [19]. A prominent candidate to realization of spin-liquid state is organic compound κ-$(ET)_2Cu_2(CN)_3$ [20]. Another class of systems which show a non-collinear (helimagnetic) order are rare-earth metals [21].

Previous investigations of classical frustrated and non-collinear antiferromagnets mainly concentrated on the critical behaviour near the magnetic phase transition. The nonlinear sigma (NLσ) model [6,7] and Landau–Ginzburg–Wilson (LGW) approach [5,22] were applied to this problem. These approaches predict critical exponents which are different from those of the standard $O(3)$ universality class. Recent investigations within the non-perturbative renormalization group approach predict, however, a first-order phase transition in three dimensions [8]. The temperature properties near the quantum first-order transition in two-dimensional (2D) frustrated antiferromagnets have been also recently investigated [23].

The consideration of physical systems requires a description of a broad control parameter range not too close to QPT. In this case one can also differentiate the high- and low-energy degrees of freedom. While the latter correspond to “slow” degrees of freedom and can be described by a continuum model, the former, “fast” degrees of freedom, can be absorbed into renormalization of parameters of the model. Thermodynamic properties at low temperatures can be expressed in this way as universal functions of ground state parameters. The thermodynamic functions describing quantum antiferromagnets can be obtained using the above mentioned NLσ model, which corresponds to a continuum limit of the Heisenberg model in the classical case. For the collinear quantum antiferromagnets the NLσ model was first derived by Haldane in the framework of the $1/S$-expansion [24,25]. Despite the way of derivation, this model can describe both the symmetry broken and the symmetric phases.

The finite temperature properties may become non-universal in certain cases, e.g., near first order phase transitions, and also for systems described by the quantum field theories at and above their upper critical dimension. The latter theories are non-renormalizable and therefore lead to non-universal properties. Especially interesting example of such a possibility occurs when the theory contains two types of interaction terms, one of which is below and another is above their upper critical dimension. While the former interactions are infrared relevant and produce universal contributions to physical properties, the latter produce non-critical but non-universal contributions. As it is shown in this paper, such a situation is realized in frustrated spin systems, in particular TLAF.

The non-collinear magnetic order in frustrated systems is described by so-called $O(3) \otimes O(2)/O(2)$ NLσ model [26]. This model includes only terms appearing in the large-S limit and is therefore quasiclassical. In comparison to the $O(3)/O(2)$ model for the square lattice this