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Collective field formulation of the multi-species Calogero model and its duality symmetries

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Abstract

We study the collective field formulation of a restricted form of the multi-species Calogero model, in which the three-body interactions are set to zero. We show that the resulting collective field theory is invariant under certain duality transformations, which interchange, among other things, particles and antiparticles, and thus generalize the well known strong-weak coupling duality symmetry of the ordinary Calogero model. We identify all these dualities, which form an Abelian group, and study their consequences. We also study the ground state and small fluctuations around it in detail, starting with the two-species model, and then generalizing to an arbitrary number of species.

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1. Introduction

The Calogero Model (CM) [1–6] is a well-known exactly solvable many-body system, both at the classical and quantum levels. The CM and its various descendants continue to draw considerable interest due to their diverse physical applications in systems such as random matrices [7], fractional statistics [8–11], gravity and black hole physics [12,13], spin chains [14,15], soli-

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tons [16–20], 2D Yang–Mills theory [21,22], lowest Landau level (LLL) anyon models [23,24], Chern–Simons matrix model [25–27], Laughlin–Hall states [28–30], and unoriented superstrings in two dimensions [31].

Calogero's original model describes N indistinguishable particles on the line which interact through an inverse-square two-body interaction. It is well known, however, that the CM may alternatively be interpreted in terms of N free particles obeying generalized exclusion statistics [9,10,32,33].

Haldane's formulation of statistics may be extended to systems made of different species of particles, in which the interspecies statistical coupling depends on the species being coupled. This may be implemented in a multi-species generalization of the CM in which particles have different masses and different couplings to each other [34–38].

Quite a few such generalized multi-species Calogero models exist, but contrary to the original CM, knowledge about their exact solvability was rather tenuous. The recent breakthrough in this front derives from the papers [39–42]. The authors of [39] introduced deformed Calogero models, related to root systems of superalgebras, and gave effectively a proof of their integrability. In [40] they presented a more conceptual proof by using shifted super-Jack polynomials. In related developments, the authors of [41,42] introduced a supersymmetric generalizations of the CM which was based on Jacobians for the radial coordinates on certain superspaces. Both aforementioned models are closely related to the multi-family generalization of the CM introduced in [43,44].

Motivated by these developments, in the present paper we investigate the latter model in the limit in which each family contains a large number of particles. In this limit, the high-density limit, the system is amenable to large-N collective-field formulation. As is well known, the collective theory offers a continuum field-theoretic framework for studying interesting aspects of many-particle systems, somewhat analogous to the continuum hydrodynamic description of fluids. It is appropriate to mention at this point the recent review on the collective-field and other continuum approaches to the spin-Calogero–Sutherland model [45]. The collective formulation has several virtues. In the large-N limit dynamics is governed by saddle points of the effective collective action, which contains the leading quantum effects. Thus, by extremizing this action, one is able to compute the uniform ground-state collective-field configuration, as well as topological and non-topological soliton configurations, and their corresponding energy eigenvalues. By expanding around these extrema it is possible to go beyond the large-N leading order and obtain the spectrum and wave-functionals of the quadratice fluctuations around these semiclassical configurations, and also to compute the corresponding density–density correlation functions.

Beside these obvious advantages, the collective-field theory provides a natural framework for analyzing symmetries of the system which cannot be seen directly in the original (finite) N-particle quantum system. An important example in this respect is the strong-weak coupling duality symmetry of the one-family Calogero model discussed in [46]. In this paper we generalize this approach to the multi-family Calogero model. We show that the resulting collective field theory is invariant under certain duality transformations, which interchange, among other things, particles and antiparticles, and thus generalize the duality symmetry [46] of the ordinary Calogero model. We identify all these dualities, which form an Abelian group, and study their consequences. In particular, the investigations carried in this paper will enable us to find the conditions under which collective quasi-particles describing density fluctuations in F-family Calogero model can be identified with those of an effective one-family Calogero model. As a by-product, this may help to better understand the exact solvability of some of the recently proposed two-family Calogero models [39–42]. We stress that the duality relations derived and discussed