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Fermi-gas interpretation of the RSOS path representation of the superconformal unitary minimal models

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Abstract

We derive new finitized fermionic characters for the superconformal unitary minimal models by interpreting the RSOS configuration sums as Fermi-gas partition functions. This extends to the supersymmetric case the method introduced by Warnaar for the Virasoro unitary minimal models. The key point in this construction is the proper identification of Fermi-type charged particles in terms of the path's peaks. For this, an instrumental preliminary step is the adaptation to the superconformal case of the operator description of the usual RSOS paths introduced recently.

1. Introduction

The solution of the Andrews–Baxter–Forrester and the Forrester–Baxter restricted-solid-on-solid (RSOS) models [2,17] by the corner-transfer matrix method leads to an expression for the local state probability of the order variable in terms of a configuration sum. Each such configuration sum provides thus a finitization of the character of an irreducible module of the corresponding minimal model.[1] In this description, every state is represented by a particular

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[1] These statistical models are related to the minimal models [19,27,28]. More precisely, in their scaling limit, the statistical models correspond to conformal field theories only at criticality. The minimal models are associated to the transition from regime III to IV. But even off-criticality, the configuration sums describe conformal characters [10–12]. This re-

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configuration [11,26]. Each configuration is naturally interpreted as a lattice path [15,33]. The finitization parameter is the path length L. The usual Virasoro characters are recovered in the limit $L \to \infty$.

The path description is combinatorial: counting the paths gives directly the character without subtraction. This provides thus a royal road for the derivation of fermionic expressions (cf. [23, 24,26]) of the characters.

Various lines of attack for deriving fermionic formulae from configuration sums have been proposed. In a seminal contribution, Melzer [26] has conjectured many finitized fermionic expressions for the unitary minimal models (extending significantly the original conjectures of [24]). The first few conjectured positive multiple-sum formulae were proved by demonstrating that they satisfy the same recurrence relations that characterize the corresponding configuration sums. This method of proof has been generalized and made into a powerful technique (under the name of telescopic expansion method) in [5], where many fermionic formulae (those for modules (r,s) with $s=1$) for all Virasoro unitary models are demonstrated. (All cases are covered in [29].) Note however that this telescopic expansion method is essentially a verification tool that requires a candidate expression for the fermionic form. But in [5], the fermionic expressions are presented as natural q-deformations of state counting problems for a specific (RSOS motivated) truncation of the space of states of the thermodynamic limit of the quantum XXZ spin chain [4]. This observation has proved to be fruitful. In [6] many new fermionic formulae (i.e., for all minimal models) are conjectured, all motivated by this counting procedure. These were successively generalized and proved in [7] by verifying that they satisfy RSOS-type recurrence relations. Note however that these expressions are related to paths in a very indirect way.

A frontal attack of the difficult combinatorics of the paths for the generic Forrester–Baxter models is considered in [14–16,36] (albeit using variables that are characteristics of the XXZ spectrum analysis and whose path interpretation is not immediate). The key preliminary step is the discovery of a new (manifestly positive definite) characterization of the weight of a Forrester–Baxter RSOS path (cf. Appendix A of [15]). From then on, the strategy followed in these works is to describe a generic path pertaining to the (finitized version of the) minimal model $\mathcal{M}(p',p)$ in terms of successive transformations acting on the unique and trivial path for the (formal) $\mathcal{M}(1,3)$ model. This relies on two explicit combinatorial transformations defined directly on the paths: a Bressoud-type transformation [9] that relates paths within a family defined by $\mathcal{M}(p',p+kp')$ for different values of $k \geqslant 1$, and a duality transformation [6] that relates $\mathcal{M}(p',p)$ to $\mathcal{M}(p-p',p)$. Although these basic transformations are quite intuitive, the resulting construction turns out to be technically rather involved. Nevertheless, all characters for all the minimal models have been written in fermionic form along this line [36].

For the unitary models, the combinatorics of the RSOS paths is considerably simplified. In that case, Warnaar has shown that a configuration sum can be regarded as a (grand-canonical) partition function of a one-dimensional gas of Fermi-type particles subject to restriction rules [33,34]. This method leads to the fermionic expression of the characters in a simple, direct and totally constructive way. Moreover, in this simpler context, the procedure can be formulated in terms of variables that have a clear path interpretation.[2]

markable off-critical relationship is explained in [30,31]. In this way, the characters of the minimal models $\mathcal{M}(p',p)$ are related to the configuration sums in regime III.

[2] For completeness, it should be added that a Fermi-gas description has been obtained also for the $\mathcal{M}(2,p)$ models in [32], but not directly from RSOS paths. Moreover, we have shown recently that the non-unitary minimal models of the type $\mathcal{M}(k+1,2k+3)$ do have a path representation similar to that of the unitary minimal models and thus an analogous