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Form factors and correlation functions of an interacting spinless fermion model

Kohei Motegi *, Kazumitsu Sakai

Institute of physics, University of Tokyo, Komaba 3-8-1, Meguro-ku, Tokyo 153-8902, Japan

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Abstract

Introducing the fermionic R-operator and solutions of the inverse scattering problem for local fermion operators, we derive a multiple integral representation for zero-temperature correlation functions of a one-dimensional interacting spinless fermion model. Correlation functions particularly considered are the one-particle Green's function and the density–density correlation function both for any interaction strength and for arbitrary particle densities. In particular for the free fermion model, our formulae reproduce the known exact results. Form factors of local fermion operators are also calculated for a finite system.

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1. Introduction

The evaluation of correlation functions has been one of the challenging problems in research on quantum many-body systems in one dimension, since most of the intriguing phenomena induced by underlying strong quantum fluctuations are theoretically described through correlation functions. In these systems, it is widely known that the existence of models which are solvable by means of Bethe ansatz (see [1,2] for example). Though the exact computation of correlation functions, of course, is still tremendously difficult even in such models, several analytical meth-

* Corresponding author.
E-mail addresses: motegi@gokutan.c.u-tokyo.ac.jp (K. Motegi), sakai@gokutan.c.u-tokyo.ac.jp (K. Sakai).

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ods have been recently developed to derive manageable expressions for correlation functions, especially in the spin-$1/2$ Heisenberg XXZ chain.

In 1990s, correlation functions of the spin-$1/2$ XXZ chain at zero temperature and for zero magnetic field have been expressed as multiple integral forms derived by the q-vertex operator approach [3–5]. An alternative method combining the algebraic Bethe ansatz with solutions to the quantum inverse problem has been provided for the XXZ chain in arbitrary magnetic fields [6–8] (see also [9] for a recent review). This method can be generalized to the finite-temperature and/or the time-dependent correlation functions [10–14].

In general, one-dimensional (1D) quantum spin systems are mapped to 1D fermion systems through the Jordan–Wigner transformation. For the spin-$1/2$ XXZ chain, the corresponding system is a spinless fermion model with the nearest neighbor hopping and interaction. In the thermodynamic limit, the bulk quantities in the XXZ chain are exactly the same as those in the spinless fermion model. Namely, in the 1D quantum systems, the difference of the statistics between the spin and the fermion does not show up, as long as we concentrate on their bulk quantities.

The situation, however, is radically changed when the quantities accompanying a change of the number of particles are considered. For instance, let us consider the (equal-time) one-particle Green's function $\langle c_1 c_{m+1}^{\dagger} \rangle$ for the spinless fermion model and the transverse spin–spin correlation function $\langle \sigma_1^{+} \sigma_{m+1}^{-} \rangle$ for the XXZ chain, which are intuitively the same. In fact, due to the difference of the statistics, or equivalently the non-locality of the Jordan–Wigner transformation, both the correlation functions exhibit completely different behavior. For instance, one observes an oscillatory behavior for the one-particle Green's function (referred to as the k_{F}-oscillation, where k_{F} is the Fermi momentum), which is peculiar to the fermion systems. In contrast, for the transverse spin–spin correlation function, such an oscillatory behavior does not appear.[1]

As mentioned above, exact expressions for the correlation functions of the XXZ chain have already been proposed in the form of multiple integral representations. Unfortunately, once the Jordan–Wigner transformation is performed and the XXZ chain is considered instead of the spinless fermion model, it is difficult to trace the difference of the statistics in the framework of multiple integral representations. Namely, to derive a manageable expression of correlation functions for the spinless fermion model, we must directly treat the fermion system from the very beginning.

In this paper, introducing the fermionic R-operator [15] which acts on the fermion Fock space, we directly treat the spinless fermion model without mapping to the XXZ chain. Combining the method provided in the XXZ chain [7] with solutions to the inverse scattering problem of local fermion operators [16], we derive a multiple integral representing the equal-time one-particle Green's function and the density–density correlation function at zero temperature both for any interaction strengths and for arbitrary particle densities. Our formulae reproduce the known results for the free fermion model. In addition to the correlation functions, we also compute form factors for local fermion operators, which might be useful for systematic evaluations of the spectral functions for the spinless fermion model.

This paper is organized as follows. In the subsequent section, we introduce the fermionic R-operator and the transfer operator constructed by the R-operator. Then we briefly review the algebraic Bethe ansatz for the spinless fermion model. The scalar product of a Bethe state with

[1] In fact, there could exist an oscillatory behavior in the transverse spin–spin correlation function, which, however, can be eliminated by a gauge transformation.