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Constructing the AdS/CFT dressing factor

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Abstract

We prove the universality of the Hernandez–Lopez phase by deriving it from first principles. We find a very simple integral representation for the phase and discuss its possible origin from a nested Bethe ansatz structure. Hopefully, the same kind of derivation could be used to constrain higher orders of the full quantum dressing factor.

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1. Introduction

Bethe ansatz equations, first written by Bethe in the study of one-dimensional metals [1], seem to be a key ingredient in the AdS/CFT duality [2] between $\mathcal{N} = 4$ SYM and type IIB superstring theory on $AdS_5 \times S^5$.

The $\mathcal{N} = 4$ SYM dilatation operator in the planar limit can be perturbatively computed in powers of the 't Hooft coupling λ. In the seminal work of Minahan and Zarembo [3] it was shown that the 1-loop dilatation operator acts on the six real scalars of the theory exactly like an

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integrable $SO(6)$ spin chain Hamiltonian. Restricting ourselves to two complex scalars we obtain the same Hamiltonian considered by Bethe for the Heisemberg spin chain. The eigenstates are K magnon states with momentum and energy parameterized by the roots u_i which satisfy Bethe equations

$$\left(\frac{u_i + i/2}{u_i - i/2}\right)^L = \prod_{j\neq i}^{K} \frac{u_i - u_j + i}{u_i - u_j - i}.$$

The full $\mathcal{N} = 4$ 1-loop dilatation operator [4] is also given by an integrable Hamiltonian whose spectrum is dictated by a system of seven Bethe equations [5], corresponding to the seven nodes of the $psu(2, 2|4)$ Dynkin diagram. In [6] the all loop version of the Bethe equation for the $SU(2)$ sector was conjectured to be

$$\left(\frac{y_i^+}{y_i^-}\right)^L = \prod_{j\neq i}^{K} \frac{u_i - u_j + i}{u_i - u_j - i}, \tag{1}$$

where $y_j(u_j)$ and $y_j^{\pm}(u_j)$ are given by

$$y + \frac{1}{y} = \frac{4\pi}{\sqrt{\lambda}} u, \qquad y^{\pm} + \frac{1}{y^{\pm}} = \frac{4\pi}{\sqrt{\lambda}}\left(u \pm \frac{i}{2}\right).$$

On the other hand, for the same sector but from the string side of the correspondence, a map between classical string solutions and Riemann surfaces was proposed [7]. The resemblance between the cuts connecting the different sheets of these Riemann surfaces and the distribution of roots of the Bethe equations in the scaling limit seemed to indicate that the former could be the continuous limit of some quantum string Bethe ansatz. In [8] these equations were proposed to be

$$\left(\frac{y_i^+}{y_i^-}\right)^L = \prod_{j\neq i}^{K} \frac{u_i - u_j + i}{u_i - u_j - i} \sigma_{\mathrm{AFS}}^2(u_i, u_j), \tag{2}$$

where

$$\sigma_{\mathrm{AFS}}(u_i, u_j) = \frac{1 - 1/(y_j^+ y_i^-)}{1 - 1/(y_j^- y_i^+)} \left(\frac{y_j^- y_i^- - 1}{y_j^- y_i^+ - 1} \frac{y_j^+ y_i^+ - 1}{y_j^+ y_i^- - 1}\right)^{i(u_j - u_i)}. \tag{3}$$

The striking similarity between (1) and (2) naturally led to the proposal that both sides of the correspondence would be described by the same equation which a scalar factor σ^2 interpolating from σ_{AFS}^2 for large 't Hooft coupling to 1 for small λ.

In [9] Beisert and Staudacher (BS) conjectured the all-loop Bethe equations for the full $PSU(2, 2|4)$ group and in [10] these equations were brought to firmer ground. As before, one of the main tools used to guess the form of these equations was the existence of the classical algebraic curve for the full superstring on $AdS_5 \times S^5$ [21]. The seven equations for the seven types