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Geometric transitions and integrable systems

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Abstract

We consider **B**-model large N duality for a new class of noncompact Calabi–Yau spaces modeled on the neighborhood of a ruled surface in a Calabi–Yau threefold. The closed string side of the transition is governed at genus zero by an A_1 Hitchin integrable system on a genus g Riemann surface Σ. The open string side is described by a holomorphic Chern–Simons theory which reduces to a generalized matrix model in which the eigenvalues lie on the compact Riemann surface Σ. We show that the large N planar limit of the generalized matrix model is governed by the same A_1 Hitchin system therefore proving genus zero large N duality for this class of transitions.

1. Introduction

Large N duality [1] has been at the center of many recent developments in topological string theory. In particular **B**-model transitions [2,3] have revealed a fascinating interplay of random matrix models, integrable systems and Calabi–Yau geometry.

In this paper we generalize the results of [2,3] reviewed in section two to a new class of conifold transitions among noncompact Calabi–Yau threefolds. As explained in section three,

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the starting point of our construction is a configuration

$$\begin{array}{ccc} \tilde{\boldsymbol{S}} & \subset & \tilde{\boldsymbol{M}} \\ \downarrow & & \downarrow \\ \boldsymbol{S} & \subset & \boldsymbol{M} \subset \boldsymbol{L} \end{array}$$

of moduli spaces which generalizes the essential geometric features of the local transitions studied in [2]. Here $\boldsymbol{L}$ is a component of the moduli space of projective or quasi-projective Calabi–Yau threefolds and $\boldsymbol{M}$ is a subspace of $\boldsymbol{L}$ parameterizing Calabi–Yau manifolds with isolated conifold singularities which admit a (quasi-)projective small resolution. The deepest stratum $\boldsymbol{S}$, which is a key element of the construction, parameterizes Calabi–Yau spaces with a genus g curve Σ of A_1 singularities. The spaces $\tilde{\boldsymbol{S}}$, $\tilde{\boldsymbol{M}}$ are moduli spaces of the resolution of Calabi–Yau spaces in $\boldsymbol{S}$ and $\boldsymbol{M}$, respectively. Such geometric structures have been considered before in the physics literature [4–6] in relation to $N = 2$ gauge theories and open string superpotentials. Here we will show that they play a key role in **B**-model geometric transitions.

Our main construction is carried out in section four. We consider noncompact Calabi–Yau spaces fibered by affine quadrics over a fixed genus g curve Σ. A special feature of this model is that the moduli spaces $\tilde{\boldsymbol{M}}$, $\boldsymbol{L}$ are isomorphic to the total spaces of vector bundles over $\boldsymbol{S}$.

Large N duality is an equivalence between **B**-type open-closed topological strings on a resolved threefold corresponding to a point in $\tilde{\boldsymbol{M}}$ and closed topological strings on a generic threefold in $\boldsymbol{L}$. In the present paper we establish this result for genus zero topological amplitudes in the geometric framework described above. The proof involves two parts.

The genus zero dynamics for closed **B**-topological strings on Calabi–Yau spaces is usually encoded in the intermediate Jacobian fibration over the moduli space, which supports an integrable system structure [7,8]. In our case we show that the relevant integrable system for a family of threefolds parameterized by a normal slice to $\boldsymbol{S}$ in $\boldsymbol{L}$ is the A_1 Hitchin integrable system. In particular, the normal slice $\boldsymbol{L}_s$ at a point $s \in \boldsymbol{S}$ is isomorphic to the space of quadratic differentials on Σ, which is the base of the Hitchin systems. This follows from a structure result for the intermediate Jacobians of the Calabi–Yau threefolds in $\boldsymbol{L}$ proved in section five.

The second part of the proof is more physical in nature and involves **B**-topological open string dynamics on a small resolution parameterized by a generic point in $\tilde{\boldsymbol{M}}$. In section six we construct the holomorphic Chern–Simons theory which captures open string target space dynamics using the formalism of D-brane categories. Then we argue that the holomorphic Chern–Simons functional integral reduces to a finite-dimensional integral on a real cycle in the product $\mathrm{Sym}^N(\Sigma) \times \mathrm{Sym}^N(\Sigma)$. This can be regarded as a generalized matrix model in which the eigenvalues are parameterized by a compact Riemann surface. The final result of this section is that the large N planar limit of this generalized matrix model is captured by the same A_1 Hitchin system that was found in section five. This concludes the physical proof of genus zero large N duality.

2. Review of Dijkgraaf–Vafa transitions

In this section we will review large N duality for a class of geometric transitions among noncompact Calabi–Yau threefolds first studied in [2]. Adopting the common terminology in the physics literature, for us a geometric transition will be an extremal transition connecting two different components of a moduli space of Calabi–Yau threefolds through a degeneration. The degenerations usually considered in this context are nodal Calabi–Yau threefolds with isolated