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Ghost conditions for Gauss–Bonnet cosmologies

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Abstract

We investigate the stability against inhomogeneous perturbations and the appearance of ghost modes in Gauss–Bonnet gravitational theories with a non-minimally coupled scalar field, which can be regarded as either the dilaton or a compactification modulus in the context of string theory. Through cosmological linear perturbations we extract four no-ghost and two sub-luminal constraint equations, written in terms of background quantities, which must be satisfied for consistency. We also argue that, for a general action with quadratic Riemann invariants, homogeneous and inhomogeneous perturbations are, in general, inequivalent, and that attractors in the phase space can have ghosts. These results are then generalized to a two-field configuration. Single-field models as candidates for dark energy are explored numerically and severe bounds on the parameter space of initial conditions are placed. A number of cases proposed in the literature are tested and most of them are found to be unstable or observationally unviable.

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1. Introduction

General Relativity (GR) is a very accurate and successful theory of classical gravity which would be desirable to embed in a more fundamental theory. The formulation of a quantum theory

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of gravity has proven to be a very difficult task and, so far, string theory is the only framework (together with loop quantum gravity) within which gravity can be incorporated at quantum level in a consistent way. Therefore it is worth exploring low-energy aspects of gravity considering, at the same time, leading-order string corrections. This leads us to the study of low-energy actions that contain a Gauss–Bonnet (GB) term, which is a particular combination of quadratic Riemann invariants.

One of the most striking peculiarities of string theory is the fact that all couplings, including those to GB terms, are given dynamically, i.e. as vacuum expectation values of fields (called, in general, moduli). This is a feature which makes the study of the evolution of the system of gravity plus moduli fields particularly interesting, as done for a Friedmann–Robertson–Walker (FRW) background in [1]. In four dimensions, the GB term is topological and does not contribute to the dynamics. Hence it is necessary for it to have a nonconstant, moduli-dependent coupling if we want it to play a nontrivial role, unless one appeals to a Lorentz-violating configuration such as the braneworld.

In this article we deal with the cosmological study of GB gravity and focus on a usually overlooked aspect, namely the possibility of having ghosts and other quantum instabilities in the model. A ghost is, by definition, a field whose kinetic term in the action is unbounded from below (roughly speaking, it has the 'wrong sign'), which implies both a macroscopic instability which, if not healed, would lead to a breakdown of the theory, and a violation of unitarity.

It is of utmost importance to realise that the presence and nature of ghost modes for a covariant Lagrangian depends on which classical background the theory is assumed to live in. For instance, in string theory the action is naturally expanded around a Minkowski target space. Ghosts in minimally coupled GB and higher-derivative gravities in *constant curvature* backgrounds were studied in [2–6]. Those approaches include both the Minkowski (first explored in [7]) and de Sitter (dS) cases, and infer the absence of spin-2 ghosts in actions whose higher-order term is a generic function of the GB combination.

One might be tempted to assume that, since de Sitter is a ghost-free vacuum of the GB theory, any cosmological model with such Lagrangian does not suffer from quantum instabilities. However, the FRW curvature invariant R is not constant and the methods used in the above-mentioned papers cannot be applied any longer. Some works have pioneered this issue by studying cosmological (i.e. FRW) perturbations for non-minimally coupled actions [8–10], albeit the ghost problem is not explicitly addressed there.

In analogy to the situation when one considers scalar–tensor theories of gravity, where the absence of spin-2 ghosts require constraints on the vacuum expectation value of the scalar (see [11,12]), we have extracted a number of constraints on the field-dependent couplings of a GB model with single scalar field in order to avoid the appearance of instabilities. These are obtained by computing the gravitational perturbations about a FRW background and studying the scalar, vector and tensor contributions separately, as done also in [8] and subsequent papers. We also imposed that perturbation modes do not propagate faster than light.

We have then applied the obtained results to different single-field models in literature [1,13,14], whose stability was studied only in phase space, that is, at the classical level (see also [15]). The *no-ghost* and *sub-luminal* constraints are here regarded as strong selection rules in the space of parameters of the theory, and we provide an algorithm necessary (but not sufficient) to guarantee quantum stability and obtain a suitable acceleration today.