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Holography for strongly coupled media

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We discuss some recent attempts to apply AdS/CFT correspondence to systems with finite temperature and chemical potential, emphasizing the hydrodynamic aspects. We also discuss the system of nonrelativistic fermions at unitarity, the Schrödinger symmetry and possible directions for constructing a holographic dual of this system. This is the write-up of lectures delivered at Cargese and TASI schools of 2010.

1. Motivation

Many problems of modern theoretical physics are related to strong coupling. One example is the problem of the hot and dense matter in QCD. The creation of hot QCD matter is the goal of relativistic heavy ion experiments, the most recent of which are RHIC and LHC. Although there are ample evidence that some form of matter with strong collective behavior is formed in ultrarelativistic heavy ion collisions, the theoretical problem of finding whether thermal equilibrium is achieved and at which temperature has still not been solved. (The problem can be made very sharp by imagining a world with very small electromagnetic fine structure constant so that nuclei can be very large. Can we make a quark gluon plasma by colliding very large nuclei at very high energy? What is the temperature of the system at thermal equilibration? We still do not have definite answer to these questions.) Assuming that system reaches equilibrium, one can ask questions about the properties of the thermal equilibrium state. While thermodynamics of the QGP at finite temperature and zero chemical potential can be studied by lattice methods, the latter becomes very inefficient in dealing with real time quantities, for example the viscosities. Current lattice methods are also incapable of treating QCD matter at finite chemical potential, a problem that hinders our understanding of the core of neutron stars.

Another example of a strong coupling problem is that of unitarity fermions (unitary Fermi gas). This system is that of nonrelativistic fermion interacting through a short-range potential fine tuned to resonance at threshold (see Section 6 for more discussion). The simplest version of the problem is the Bertsch problem: given a gas of spin-1/2 fermions, interacting with short-range interaction fine tuned to unitarity (defined below in the lectures), what are the properties of the ground state? This problem has became extremely important when it became possible to realize unitarity fermions in atomic trap experiments.

Various other strong coupling problems in condensed matter physics are discussed in Subir Sachdev's lectures in this school. In these lectures, we will describe some points of contact between gauge/gravity duality and the physics of the quark gluon plasma and the unitary Fermi gas.

2. Thermal field theory

There are two main formalisms used in thermal field theory. The first formalism is the Matsubara, Euclidean formalism. It is used in lattice QCD, very convenient for thermodynamic and static quantities (like correlation length), but cannot directly address dynamic, real-time quantities. The second formalism is the real-time, close time path formalism. (For more details, see Ref. [1,2]).

In the Matsubara formalism, the theory is formulated on a Euclidean spacetime, where the time axis is compactified to an interval $0 < \tau <$

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$\beta = 1/T$. In the close-time-path formalism, one makes a detour into real time, as in Fig. 1.

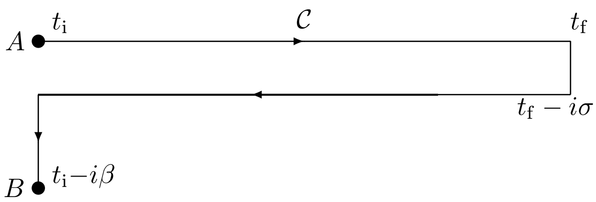

Figure 1. The close time path contour

One can turn on source on the upper and lower parts of the contour, J_1 and J_2. The partition function of field theory now is a functional of both J_1 and J_2, $Z = Z[J_1, J_2]$, and and derivatives of $\log Z$ with respect to J gives a 2×2 matrix propagators G_{ab}, where $a, b = 1, 2$. Changing σ rescales the off-diagonal elements by a trivial factor,

$$G_{12}(\omega, q) = e^{\sigma\omega} G_{12}^{\sigma=0}(\omega, q), \quad (1)$$

$$G_{21}(\omega, q) = e^{-\sigma\omega} G_{21}^{\sigma=0}(\omega, q). \quad (2)$$

For $\sigma = 0$, the propagators G_{ab} include path-ordered, reversed path-ordered, and Wightmann Green's functions. They are related by

$$G_{11} + G_{22} = G_{12} + G_{21}, \quad (3)$$

but the choice $\sigma = \beta/2$ leads to symmetric 2×2 propagator matrix: $G_{12} = G_{21}$. This choice of the σ is most natural for holography, as we will see.

From the point of view of the CTP formalism, putting our system in an external source J corresponds to having, in the $\sigma = 0$ choice of the contour, $J_1 = J_2 = J$. The expectation value of the operator ϕ at a point x on an upper contour is given by an integral over the whole contour, which can be written as

$$\langle \phi_1(x)\rangle = -\int dy\, (G_{11}(x-y)J(y) - G_{12}(x-y)J(y)), \quad \sigma = 0. \quad (4)$$

Define the retarded propagator $G_R = G_{11} - G_{12}$ ($\sigma = 0$). The retarded propagator governs the response of a system to a small external perturbation:

$$\langle \phi(x)\rangle = -\int dy\, G_R(x-y)J(y). \quad (5)$$

On the other hand, for the symmetric choice $\sigma = \beta/2$, $G_R = G_{11} - e^{-\beta\omega/2}G_{12}$.

Normally, the computations of thermal Green's function rely on summing Feynman diagrams. The set of Feynman diagrams that one has to sum in order to compute, say, the viscosity, can be quite complicated [3]. In the low-momentum limit, however, the forms of many correlation functions are simple and are dictated by an effective theory—hydrodynamics.

3. Hydrodynamics

Consider an interacting quantum field theory at finite temperature. One can visualize such a system as a collection of particles (or quasiparticles), moving with random velocities and colliding with each other from time to time. Such a picture is too simplistic for a strongly interacting system (with no discernible quasiparticles) but it does tell us that there is an important length scale in the problem—the mean free path, which is the length which a particle travels before colliding with other particles.

Hydrodynamics can be thought of as an effective theory describing the dynamics of a finite-temperature system at distance scales much larger than the mean free path. By definition the degrees of freedom entering hydrodynamics have to have relaxation time much larger than the mean free time. Such modes include

- Density of conserved quantities. Consider, for example, the QCD plasma, and imagine a perturbation of the system where there is a net excess of charge in a volume with size $L \gg \ell$. If one waits a long time this lump of excess charge will disappear, with the charge now distributing uniformly over the whole volume. However, since charge is conserved, causality implies that the time