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Nuclear Physics B (Proc. Suppl.) 216 (2011) 194–210
NUCLEAR PHYSICS B PROCEEDINGS SUPPLEMENTS
www.elsevier.com/locate/npbps

Lectures on the Kerr/CFT Correspondence

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We give a short introduction, beginning with the Kerr geometry itself, to the basic results, motivation, open problems and future directions of the Kerr/CFT correspondence.

1. Introduction

In the early 1970's, work by Bekenstein, Carter, Christodolou, Hawking, and many others [1–7] raised profound puzzles about the nature of black holes. One striking such puzzle was that, while macroscopic arguments gave the entropy of a black hole as one quarter of its event horizon area:

$$S = \frac{A}{4\hbar G}, \tag{1}$$

at the time no microscopic accounting for this entropy was known. It seemed imperative that we should be able to account for the black hole entropy microscopically, just as had been done in the nineteenth century for gases and liquids. Without such a microstate description, we would seem to run into serious contradictions.

This problem remained largely unsolved for more than 20 years. Then in the mid 90's string theory was used [8] to explicitly identify the missing microscopic degrees of freedom for a very particular kind of black hole. This calculation depended on many specific details of string theory. At the end of a rather lengthy computation involving numerous factors of 2, π etc., the Bekenstein-Hawking result (1) was reproduced by counting microstates. At the time, it was argued that this precise match provided indirect evidence for string theory as the correct theory of nature.

However, about a year later, it was shown [9] that in fact, any consistent, unitary quantum theory of gravity containing those particular black holes - characterized by a near-horizon region with an AdS_3 factor - as solutions must reproduce the entropy in essentially the same way. The specific details of string theory as the microscopic UV completion were not necessary. Rather, the key ingredient followed from the analysis done by Brown and Henneaux [10] in the 80's: if we find a consistent completion of quantum gravity on AdS_3 it has to be described by a 2D conformal field theory due to purely symmetry considerations. Thus, the detailed matching of the factors of 2 and π was not really a consequence of string theory but rather, it simply had to follow because string theory is a consistent theory of quantum gravity. Any other consistent theory must by necessity also reproduce the same result in the same manner.[1]

Since then, we have slowly but surely been progressing in our understanding of the relation between black holes and 2D CFTs. We started with 5D supersymmetric black holes, then proceeded to partially supersymmetric and then to the 3D nonsupersymmetric black holes with near-horizon AdS_3 geometry. Recently, our understanding has finally evolved to up the point where we can understand something about 4D Kerr black holes that we see up in the sky.

The work we are going to discuss is heavily informed by string theory, but none of it relies on the conjecture that string theory is the actual

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[1] The other side of the coin here is that these general arguments imply that any consistent quantum theory of gravity must, on an AdS$_3$ background, behave a lot like string theory – so much so that we might reasonably call it string theory!

0920-5632/$ – see front matter
doi:10.1016/j.nuclphysbps.2011.04.155

theory of nature, or on the stringy realizations of the AdS/CFT correspondence. Instead, all of our arguments follow from careful study of the diffeomorphism group together with some basic consistency assumptions, and do not involve any details of Planck scale physics. Indeed it would be very strange if the universal area-entropy law somehow depended on the exact microscopic details of how quantum gravity is completed in the UV!

To emphasize this point further, let us draw an analogy of the current efforts with the work of Boltzmann in the 19th century. At that time thermodynamics was understood, but people did not know much about atoms and molecules. Boltzmann wanted to explain the laws of thermodynamics by applying statistical, probabilistic reasoning to the fundamental constituents (degrees of freedom) of gases and liquids. However, he encountered a UV divergence: if a gas is treated as a continuous medium, then it has infinitely many degrees of freedom because of the existence of arbitrarily short wavelength modes. Any attempt to derive the thermodynamics of gases by applying statistical reasoning to a theory of a continuous medium, will hit the so called Rayleigh ultraviolet catastrophe in which all energy is eventually sucked into the UV modes. To avoid this problem, a consistent UV cutoff is needed. People were already talking at that time about atoms and molecules, so Boltzmann assumed that there was some theory of atoms, i.e. he assumed that there was a consistent UV cutoff for gases and liquids. He did not at all need to know what the details of this atomic cutoff were; in fact, the periodic table was not discovered until more than fifty years later. Boltzmann's mere assumption that there existed a UV cutoff at some energy scale was sufficient to derive the universal laws of thermodynamics from statistical reasoning. Of course, having a detailed microscopic theory can provide more information; for example if one wants to compute the heat capacity from first principles, one needs a detailed UV completion (that is, the actual quantum theory of atoms and molecules).

We might hope that a similar story holds for black holes. We should not need to know all the details of string theory at scales of order 10^{-38} km in order to understand why the area law (1) applies to the black hole Sagittarius A* in the center of our galaxy which is 10^7 km across! We should be able to understand the area law just from the assumption that quantum gravity has *some* consistent UV completion.

The stringy microscopic entropy analysis in [8] was akin to first computing the periodic table and then using it to compute the laws of thermodynamics. In this stringy black hole computation we had far more information than was necessary to get the area law: we had huge sets of numbers for degeneracies at any level. Only a tiny part of this information turns out to be universal. We are going to see in these lectures that this tiny universal part can be understood using universal reasoning and no assumptions about Planck scale cutoffs. This is exactly as it should be.

In these lectures we will encounter another much-studied object in theoretical physics which has a lot of universal behavior: 2D conformal field theories. Many features we know of 2D CFTs are independent of the details of a given CFT. Indeed, we will find a striking match -going far beyond the entropy formula (1) - between the universal properties of 2D conformal field theories and those of black holes.

The plan for the rest of the lectures is the following: we will start with a review of the Kerr geometry, including the Near-Horizon Extreme Kerr (NHEK) geometry. Then we will cover the asymptotic symmetry group, boundary conditions for the NHEK geometry, the CFT description of a quantum theory of gravity in NHEK and the surprising evidence for hidden conformal symmetries far from extremality. We will close with a discussion of open problems and future directions.

2. Kerr geometry

2.1. The Kerr solution

There is a famous quote from Chandrasekhar [11]

"*.... Kerr's solution has also surpassing theoretical interest: it has many properties that have the aura of the miraculous about them.* "