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Nuclear Physics B (Proc. Suppl.) 192–193 (2009) 193–196
NUCLEAR PHYSICS B
PROCEEDINGS SUPPLEMENTS
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# Real-time gauge/gravity duality and ingoing boundary conditions

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In Lorentzian gauge/gravity duality, a proper understanding of initial conditions is essential. I discuss the precise relation between purely ingoing conditions at the horizon for bulk fields and retarded boundary correlation functions, as well as the generalization to higher-point functions. Some open questions can be answered only within the recently developed framework of [1,2].

## Introduction

The gauge/gravity duality [3] is by now firmly rooted in a well-developed dictionary between the gauge theory and the gravity side. Within the supergravity approximation, most entries in the dictionary can be conveniently summarized in the familiar formula

$$Z_{\rm qft}[J] = \exp\big( - S_{\rm sugra}[J]\big)\,, \qquad (1)$$

where $J$ represents both QFT sources as well as boundary conditions for the supergravity fields.

However, (1) is really valid only in imaginary time. The real-time dictionary is necessarily more involved than the continuation of (1), since one has to specify both initial and final QFT states as well as initial and final supergravity data. Historically, this complication was overcome by imposing initial and final boundary conditions that were motivated purely within supergravity. For example, the authors of [4] used a black hole argument to state that retarded real-time thermal correlation functions can be obtained by using 'purely ingoing' boundary conditions for the supergravity fields at a bulk horizon. Subsequently, in [5] these purely ingoing conditions were tied to 'natural' boundary conditions in the same way as retarded and time-ordered correlation functions are related in field theory. The prescription of [4,5] turned out to be very successful and is by now widely used.

However, in my opinion a complete first-principles derivation of this prescription has been missing and some questions remained unanswered. For example, in general one expects the field theory state (or rather ensemble) to determine all the initial conditions, including any boundary conditions for fluctuations at the horizon. Why does this not seem to be the case here? Could we not change the state somewhat and obtain different ('non-natural') boundary conditions? And if the prescription is related to an on-shell action like (1) as suggested in [5], why can we ignore surface contributions from the initial and final boundaries to this action?

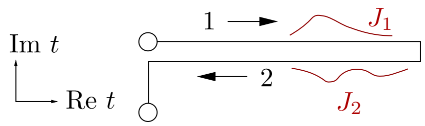


Figure 1. A real-time thermal contour in the complex time plane. The circles should be identified. The two Lorentzian segments are labelled 1 and 2 on which we have sources $J_1$ and $J_2$, respectively.

In [1,2], see also [6], a real-time gauge/gravity dictionary was developed from first principles. The aim of this note is to show that this new dictionary reproduces almost precisely the recipe of [4] and answers all the questions raised in the previous paragraph as well.

## A complete dictionary

Consider a field theory at finite temperature $T = 1/\beta$. The dynamics of the corresponding gas or plasma is described by *real-time thermal correlation functions.* These correlators can be obtained [7] from a path integral along a contour in the complex time plane as sketched in Fig. 1, with sources $J_1$ and $J_2$ (for an operator $\mathcal{O}$) on the two horizontal segments of the contour. We


doi:10.1016/j.nuclphysbps.2009.07.078

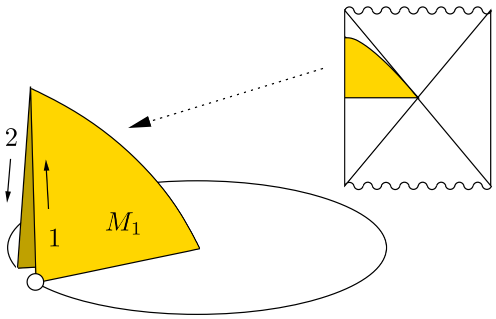


Figure 2. The Euclidean segment of the contour is filled in with a disk; the two Lorentzian segments with two copies of a part of an eternal black hole spacetime (shaded).

will in particular compute the *retarded* correlator $i\Delta_R(x,x') = \theta(t-t')\langle[\mathcal{O}(x),\mathcal{O}(x')]\rangle$, which is obtained by setting $J_1 = J_2 \equiv J$ and expanding the one-point function of $\mathcal{O}$ to first order in $J$:

$$\delta_J\langle\mathcal{O}(x)\rangle = \int d^dx' \Delta_R(x,x')J(x') + \dots \quad (2)$$

We will use this equation for $\Delta_R$ below.

Let us now turn to the real-time gauge/gravity prescription of [1,2]. It instructs us to fill in the *entire* field theory contour with bulk spacetimes. Consider first the vertical segment in Fig. 1 and suppose that it can be filled in with a Euclidean black hole solution. Topologically, this fills the imaginary time circle with a disk (plus some transverse space which is unimportant here). To add in the Lorentzian segments, we slice open the Euclidean black hole solution by making a cut in the disk, say at Euclidean time $\tau = 0$ up to the center of the disk. To the two cut surfaces we glue two copies of a segment of an eternal Lorentzian black hole solution which we will call $M_1$ and $M_2$. We finally glue $M_1$ and $M_2$ together along some late-time surface. The total space is sketched in Fig. 2.[1]

As usual, the sources $J_1, J_2$ on the boundary contour now correspond to boundary data for the supergravity fields and switching them on causes perturbations on the background of Fig. 2. These perturbations propagate from one segment to the other via the *matching conditions* of [2] that essentially guarantee $C^1$ continuity of the fields across the gluing. (The precise conditions can be derived from a saddle-point approximation.)

[1]This space differs from that of [2] only by a downward deformation of the late-time hypersurface. Such a deformation is unimportant for the boundary correlators.

**Ingoing boundary conditions**

As an example, let us consider a free bulk scalar field $\Phi$ with mass $m$ satisfying the bulk Klein-Gordon equation with general boundary data $J_1, J_2$. For brevity, we will only write down the explicit solution on the segment $M_1$.

We assume that we can use separation of variables in $t$, the angular (or other transverse) coordinates $\vec{\varphi}$ and the radial coordinate $r$. One then finds four mode solutions,

$$e^{-i\omega t}Y_l(\vec{\varphi})\phi_{\pm\pm}(\omega, l, m^2, r)\,,$$

with $Y_l$ some basis of harmonic functions on the transverse space. These modes are either purely ingoing ($\phi_{-\pm}$) or purely outgoing ($\phi_{+\pm}$); the second $\pm$ indicates the different possible analytic continuations across the horizons which we do not need here. Any solution $\Phi$ can be expanded in these modes with certain coefficients $a_{\pm\pm}$:

$$\Phi(t,\vec{\varphi},r) = \sum_l \int d\omega\, e^{-i\omega t}Y_l(\vec{\varphi})(a_{++}\phi_{++} + a_{+-}\phi_{--} + a_{-+}\phi_{-+} + a_{--}\phi_{--})\,. \quad (3)$$

Now consider the solution corresponding to a delta-function source at $(t,\vec{\varphi}) = 0$ on $\partial_r M_1$ and denote the corresponding solution on $M_1$ as $\Delta_{[11]}$. If the modes are appropriately normalized, then $\Delta_{[11]}$ has the form (3) with

$$a_{[11]++} = (1 - e^{\beta\omega})^{-1} \qquad a_{[11]+-} = 0$$
$$a_{[11]-+} = 0 \qquad a_{[11]--} = (1 - e^{-\beta\omega})^{-1}\,.$$

These $a_{[11]\pm\pm}$ are uniquely determined by demanding normalizability along the radial boundary of the entire manifold of Fig. 2 (except of course at the origin of $\partial_r M_1$), combined with the matching conditions between the segments. This solution precisely satisfies the ‘natural’ boundary conditions of [5].

Let us now move the delta-function source to the origin of $\partial_r M_2$. The perturbation propagates