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Matching Coefficients for the Strong Coupling in the Standard Model and MSSM

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In this contribution we review the status of the decoupling coefficients for α_s both in the Standard Model (SM) and its minimal supersymmetric extension, the so-called MSSM. We stress the importance of a consistent treatment, in particular within the framework of the MSSM.

1. Motivation

The strong coupling constant, α_s, constitutes a fundamental parameter in the Standard Model (SM) and thus its precise numerical value is very important for many physical predictions. An interesting property of α_s is its scale dependence, in particular its strong rise for low and its small value for high energies which make perturbative calculations within the framework of QCD possible. The scale dependence is governed by the β function. However, in order to relate α_s at two different scales it is also necessary to incorporate threshold effects of heavy quarks which is achieved with the help of the so-called matching or decoupling relations. Thus, when specifying α_s it is necessary to indicate next to the scale also the number of active flavours.

A consistent treatment of the running and decoupling procedure requires the use of $(N-1)$-loop decoupling formulae together with N-loop expressions for the β function. As far as QCD is concerned, recently the four-loop matching relations have been computed by two independent groups [1,2]. Formally this would require the knowledge of the five-loop β function, however, currently only the four-loop terms are known [3,4].

The situation is different in the context of supersymmetry. In the limit where the number of active fermions and sfermions is equal the β function up to three loops is available since already quite some time [5], however, until recently only the one-loop matching conditions have been available in the literature. This gap has been closed in Ref. [6] where two-loop decoupling relations in the framework of the MSSM have been studied.

In this contribution we provide a brief overview on the current status, both for QCD, which is treated in Section 2, and SUSY-QCD, which can be found in Section 3.

2. Decoupling in QCD

A typical problem where the question of decoupling plays a crucial role occurs when two values of α_s, defined for different number of active flavours, shall be related to each other. E.g., consider the case where $\alpha_s^{(5)}(M_Z)$ shall be computed from the knowledge of $\alpha_s^{(4)}(M_\tau)$. In this example the decoupling constant has to provide a relation between $\alpha_s^{(4)}(\mu_b)$ and $\alpha_s^{(5)}(\mu_b)$ where μ_b is the matching scale where the bottom quark is decoupled.

In a more general way we define the decoupling constant via

$$g_s^{0\prime} \;=\; \zeta_g^0 g_s^0\,, \tag{1}$$

where $g_s^0=\sqrt{4\pi\alpha_s^0}$ is the bare strong coupling. In close analogy to the renormalization procedure it is possible to relate ζ_g^0 to the decoupling constants of the ghost-gluon vertex, the gluon propagator and the ghost propagator:

$$\zeta_g^0 \;=\; \frac{\tilde\zeta_1^0}{\tilde\zeta_3^0\sqrt{\zeta_3^0}}\,. \tag{2}$$

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It is convenient to define the ζ's in a first step for bare quantities since then one-to-one relations between the N-loop contributions to the decoupling constants and the N-loop vacuum integrals can be established [7]. Sample diagrams up to four-loop order can be found in Fig. 1(a), (b) and (c). The technology up to three-loop order is well documented in the literature and publicly available programs exist (see, e.g., Refs. [8,9]). The development of the four-loop technology is quite new. The results of Ref. [1] are based on a program [10] where the so-called Laporta algorithm has been implemented in FORM [11].

Once the bare four-loop result for the decoupling relation is available it is straightforward to obtain the renormalized expression with the help of the usual renormalization constants (see, e.g., Ref. [12])

$$\zeta_g = \frac{Z_g}{Z'_g}\zeta_g^0 . \qquad (3)$$

The analytical result for ζ_g can be found in Ref. [1] where only one constant, which was denoted by X_0, has not been known in analytical form but with high numerical precision. Meanwhile also this constant is available analytically [13] with the result

$$X_0 = -384a_5 + \frac{873}{2}\zeta_5 - \frac{53}{15}\pi^4 \ln 2 - \frac{16}{3}\pi^2 \ln^2 2 + \frac{16}{5}\ln^5 2 \approx 1.808879546208334741\ldots , \qquad (4)$$

where $a_5 = \mathrm{Li}_5(1/2)$ and $\zeta_5 \approx 1.036927755\ldots$.

In order to study the numerical impact of our result we consider the evaluation of $\alpha_s^{(5)}(M_Z)$ from $\alpha_s^{(4)}(M_\tau)$, i.e. we apply our formalism to the crossing of the bottom quark threshold. In general one assumes that the value of the scale μ_b, where the matching has to be performed, is of order m_b. However, it is not determined by theory. Thus this uncertainty contributes significantly to the error of physical predictions. On general grounds one expects that while including higher order perturbative corrections the relation between $\alpha_s^{(4)}(M_\tau)$ and $\alpha_s^{(5)}(M_Z)$ becomes insensitive to the choice of the matching scale which we want to demonstrate in the following up to the five-loop order.

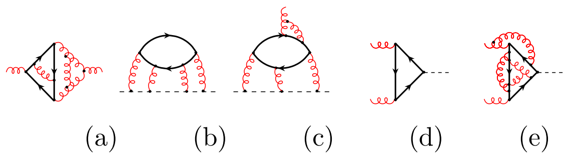

Figure 1. Sample diagrams for the gluon (a) and ghost (b) propagator and the ghost-gluon vertex (c). In (d) the lowest-order diagram is shown mediating the Higgs-gluon coupling in the SM and (e) shows an example for a five-loop diagram contributing to the coefficient function C_1.

Starting from $\alpha_s^{(4)}(M_\tau) = 0.36$ we compute in a first step $\alpha_s^{(4)}(\mu_b)$ using N-loop accuracy. Afterwards $\alpha_s^{(5)}(\mu_b)$ is obtained where for consistency the $(N-1)$-loop matching equations have to be used. Finally, we compute $\alpha_s^{(5)}(M_Z)$ using again the N-loop in the β function.

In Fig. 2 the result for $\alpha_s^{(5)}(M_Z)$ as a functions μ_b is displayed for the one- to five-loop analysis. For illustration, μ_b is varied rather extremely, by almost two orders of magnitude. While the leading-order result exhibits a strong logarithmic behaviour, the analysis is gradually getting more stable as we go to higher orders. The five-loop curve is almost flat for $\mu_b \geq 1$ GeV and demonstrates an even more stable behaviour than the four-loop analysis of Ref. [7]. It should be noted that around $\mu_b \approx 1$ GeV both the three-, four- and five-loop curves show a strong variation which can be interpreted as a sign for the breakdown of perturbation theory. Note that for this analysis the unknown five-loop coefficient of the β function is set to zero.

An interesting connection between the decoupling constant discussed so far, which constitutes a fundamental quantity of QCD, and a building block for the production and decay of a SM Higgs boson is established by the all-order low-energy theorem [7] given by

$$C_1 = -\frac{1}{2}\frac{m_t^2\partial}{\partial m_t^2}\ln\zeta_g^2 . \qquad (5)$$

Eq. (5) relates ζ_g to the coefficient function appearing in the effective Lagrangian of an intermediate-mass Higgs boson to two, three and