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Recursive equations for arbitrary scattering processes *

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The usefulness of recursive equations to compute scattering matrix elements for arbitrary processes is discussed. Explicit results at tree and one-loop order, obtained by the `HELAC/PHEGAS` package that is based on the Dyson-Schwinger recursive equations approach, are briefly presented.

1. Introduction

Recursive equations to compute scattering matrix elements have been used extensively over the last years in order to obtain results for multi-leg amplitudes. Their history started essentially with the work of Berends and Giele [1], who were able to prove the conjectured simple all-n form of Parke and Taylor [2] for the MHV amplitudes in QCD. The recognition of their usefulness has been expanded recently by the discovery of a new class of recursive equations, by Britto, Cachazo and Feng [3] and Witten [4].

In this paper we are considering the Dyson-Schwinger (DS) recursive approach [5–8], and show how this can be used as a general framework for scattering elements computation. We also present selected results for processes at tree order and at the one-loop level, obtained with the `HELAC/PHEGAS` [9] package, which is an implementation of the DS method.

2. The Dyson-Schwinger approach

The traditional representation of the scattering amplitude in terms of Feynman graphs results to a computational cost that grows like the number of those graphs, therefore as $n!$ (at tree order), where n is the number of particles involved in the scattering process.

An alternative[1] to the Feynman graph representation is provided by the Dyson-Schwinger approach [7]. Dyson-Schwinger equations express recursively the n-point Green's functions in terms of the $1-, 2-, \ldots, (n-1)$-point functions. In the framework of a theory with three- and four-point vertices the DS equations are rather simple and their diagrammatic representation is given below, for $1 \to n$ [12–17] amplitude:

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[1]For other alternatives see [10,11].

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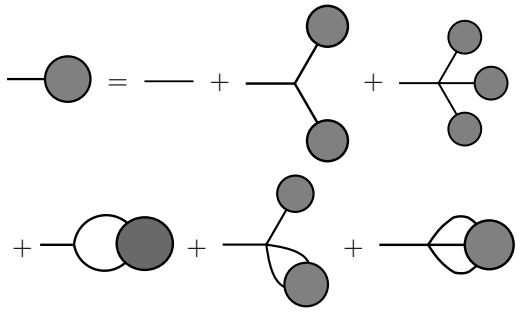

Omitting the contribution of the second line in the above formula is equivalent to restrict ourselves at tree order. In order to get an idea of the actual mathematical form of these equations, let as consider the simplest case where we are interested to "count graphs", so by dropping all propagators, couplings, wave-functions, etc, we end up with the following equation:

$$a(n) = \delta_{n,1} + \frac{1}{2!}\sum \frac{n!}{n_1!n_2!} a(n_1)a(n_2)\delta_{n_1+n_2,n}$$
$$+\frac{1}{3!}\sum \frac{n!}{n_1!n_2!n_3!} a(n_1)a(n_2)a(n_3)\delta_{n_1+n_2+n_3,n}$$

with the initial condition $a(0) = 0$; $a(n)$ is nothing more than the number of Feynman graphs, contributing to the $1 \to n$ matrix element.

The computational cost of DS equations grows like $\sim 3^n$, which essentially counts the steps used to solve the recursive equations. Obviously for large n there is a tremendous saving of computational time, compared to the $n!$ growth of the Feynman graph approach.

2.1. Color representation

Color representation or color decomposition of the amplitude is a major issue when dealing with multi-parton processes. Let us consider n-gluon scattering with external momenta $\{p_i\}_1^n$, helicities $\{\varepsilon_i\}_1^n$ and colors $\{a_i\}_1^n$ of gluons $i = 1, \ldots, n$. As is well known the total amplitude can be expressed as a sum of single trace terms [18]:

$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n) = 2ig^{n-2}$$
$$\sum_{I\in P(2,\ldots,n)} Tr(t^{a_1} t^{a_{\sigma_I(2)}} \ldots t^{a_{\sigma_I(n)}}) \mathcal{A}_I(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

where $\sigma_I(2:n)$ represent the I-th permutation of the set $\{2, \ldots, n\}$ and $Tr(t^{a_1} t^{a_{\sigma_I(2)}} \ldots t^{a_{\sigma_I(n)}})$ represents a trace of generators of the $SU(N_c)$ gauge group in the fundamental representation. For processes involving quarks a similar but much more cumbersome expression can be derived [18].

One of the most interesting aspects of this decomposition is the fact that the $\mathcal{A}_I(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$ functions (called dual, partial or color-ordered amplitudes), which contain all the kinematic information, depend on the permutation and are gauge invariant and cyclically symmetric in the momenta and helicities of gluons. The color ordered amplitudes are simpler than the full amplitude because they only receive contributions from diagrams with a particular cyclic ordering of the external gluons (planar graphs).

Of course to get the full amplitude one has to square the matrix element,

$$\sum_{\{a_i\}_1^n \{\varepsilon_i\}_1^n} |\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n)|^2$$
$$= g^{2n-4} \sum_{\varepsilon} \sum_{ij} \mathcal{A}_I \mathcal{C}_{IJ} \mathcal{A}_J^*$$

where the $(n-1)! \times (n-1)!$ dimensional color matrix can be written in the most general form as follows:

$$\mathcal{C}_{IJ} = \sum_{1\ldots N_c} Tr(t^{a_1} t^{a_{\sigma_I(2)}} \ldots t^{a_{\sigma_I(n)}}) Tr(I \leftrightarrow J)^* \tag{1}$$

There exists a much simpler approach, in fact far superior from the point of view of an automatized numerical calculation, where the matrix element is represented as follows [19,7,20,8],

$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{c_i, a_i\}_1^n) =$$
$$2ig^{n-2} \sum_{I=P(2,\ldots,n)} D_I \;\; \mathcal{A}_I(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

with c_i, a_i the color and anticolor indices for each external particle, i.e. $(c, 0)$ for quarks, $(0, a)$ for antiquarks, (c, a) for gluons and $(0, 0)$ for non-colored particles, and

$$D_I = \delta_{c_1, a_{\sigma_I(1)}} \delta_{c_2, a_{\sigma_I(2)}} \ldots \delta_{c_n, a_{\sigma_I(n)}}$$