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Two-point gauge invariant quark Green's functions with polygonal phase factor lines

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Abstract

Polygonal lines are used for the paths of the gluon field phase factors entering in the definition of gauge invariant quark Green's functions. This allows classification of the Green's functions according to the number of segments the polygonal lines contain. Functional relations are established between Green's functions with polygonal lines with different numbers of segments. An integrodifferential equation is obtained for the quark two-point Green's function with a path along a single straight line segment where the kernels are represented by a series of Wilson loop averages along polygonal contours. The equation is exactly and analytically solved in the case of two-dimensional QCD in the large-N_c limit. The solution displays generation of an infinite number of dynamical quark masses accompanied with branch point singularities that are stronger than simple poles. An approximation scheme, based on the counting of functional derivatives of Wilson loops, is proposed for the resolution of the equation in four dimensions.

Keywords: QCD, quark, gluon, Wilson loop, gauge invariant Green's function

1. Introduction

Path-dependent phase factors are the natural ingredients in gauge theories for the description of parallel transport of gauge covariant quantities in integrated form [1, 2]. They also allow for the construction of gauge invariant Green's functions, which are the adequate tools for the study of the physical properties of the theory. The Wilson loop [3], which corresponds to a phase factor along a closed contour, is used to set up a criterion for the recognition of the confinement of quarks in QCD [3, 4, 5]. Properties of Wilson loops were thoroughly studied in the past [6, 7, 8, 9, 10, 11] and applications to bound states of quarks were considered [12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

On the other hand, approaches using gauge invariant correlators meet difficulties due to the extended nature of the phase factors and could not up to now provide a complete systematic procedure of solving the theory. In spite of these difficulties, the advantages one might expect from a gauge invariant approach merit continuation of the efforts that are undertaken. Gauge invariant quantities are expected to have an infrared safe behavior, free of artificial or unphysical singularities. For this reason, they are better suited to explore the nonpertubative regime of the theory; in QCD, this mainly concerns the occurrence of confinement. Also, the knowledge of gauge invariant wave functions of bound states facilitates calculations of matrix elements of operators involving phase factors.

The present talk is a summary of recent work of the author [22, 23] trying to deduce exact integrodifferential equations for two-point quark gauge invariant Green's functions (2PQGIGF), in analogy with the Dyson-Schwinger equations of ordinary Green's functions [24, 25, 26, 27, 28]. The method of approach is based on the use of polygonal lines for the paths of the phase factors. Polygonal lines are of particular interest

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since they can be decomposed as a succession of straight line segments with mutual junction points. Straight line segments are Lorentz invariant in form and have an unambiguous limit when the two end points approach each other. Polygonal lines can be classified according to the number of segments or sides they contain, which in turn is reflected on the 2PQGIGFs.

2. Green's functions with polygonal lines

Let $U(y,x)$ be a path-ordered phase factor along an oriented straight line segment going from x to y. A displacement of one end point of the rigid segment, while the other end point is fixed, generates also a displacement of the interior points of the segment. This defines a rigid path displacement. Parametrizing the interior points of the segment with a linear parameter λ varying between 0 and 1, such that $z(\lambda) = \lambda y + (1-\lambda)x$, the rigid path derivative operations with respect to y or x yield [1, 2, 29, 30]

$$\frac{\partial U(y,x)}{\partial y^\alpha} = -igA_\alpha(y)U(y,x) + ig(y-x)^\beta \times \int_0^1 d\lambda\lambda U(y,z(\lambda))F_{\beta\alpha}(z(\lambda))U(z(\lambda),x), \qquad (1)$$

$$\frac{\partial U(y,x)}{\partial x^\alpha} = +igU(y,x)A_\alpha(x) + ig(y-x)^\beta \times \int_0^1 d\lambda(1-\lambda)U(y,z(\lambda))F_{\beta\alpha}(z(\lambda))U(z(\lambda),x), \qquad (2)$$

where A is the gluon potential, F its field strength and g the coupling constant.

In gauge invariant quantities, the end point contributions of the segments are usually cancelled by other neighboring point contributions and one remains only with the interior point contributions of the segments, represented by the integrals above. We introduce for them a shorthand notation:

$$\frac{\bar{\delta}U(y,x)}{\bar{\delta}y^{\alpha+}} = ig(y-x)^\beta \int_0^1 d\lambda\lambda U(y,z(\lambda)) \times F_{\beta\alpha}(z(\lambda))U(z(\lambda),x), \qquad (3)$$

$$\frac{\bar{\delta}U(y,x)}{\bar{\delta}x^{\alpha-}} = ig(y-x)^\beta \int_0^1 d\lambda(1-\lambda)U(y,z(\lambda)) \times F_{\beta\alpha}(z(\lambda))U(z(\lambda),x). \qquad (4)$$

The superscript + or − on the derivative variable takes account of the orientation on the segment and specifies, in the case of joined segments, the segment on which the derivative acts.

The vacuum expectation value (or vacuum average) of a Wilson loop along a contour C will be designated by $W(C)$. In the case of a polygonal contour C_n, with n segments and n junction points $x_1, x_2, \ldots, x_n$, it will be designated by W_n and represented as an exponential functional [8, 10]:

$$W_n = W(x_n, x_{n-1}, \ldots, x_1) = e^{F_n(x_n,x_{n-1},\ldots,x_1)} = e^{F_n}. \qquad (5)$$

The 2PGIQGF with a phase factor along a polygonal line composed of n segments and $(n-1)$ junction points is designated by $S_{(n)}$:

$$S_{(n)}(x,x';t_{n-1},\ldots,t_1) = -\frac{1}{N_c}\langle\overline{\psi}(x')U(x',t_{n-1}) \times U(t_{n-1},t_{n-2})\ldots U(t_1,x)\psi(x)\rangle, \qquad (6)$$

the quark fields, with mass parameter m, belonging to the fundamental representation of the color gauge group $SU(N_c)$ and the vacuum expectation value being defined in the path integral formalism. (Spinor indices are omitted and the color indices are implicitly summed.) The simplest such function is $S_{(1)}$, having a phase factor along a straight line segment:

$$S_{(1)}(x,x') \equiv S(x,x') = -\frac{1}{N_c}\langle\overline{\psi}(x')U(x',x)\psi(x)\rangle. \qquad (7)$$

(We shall generally omit the index 1 from that function.)

For the internal parts of rigid path derivatives, we have definitions of the type

$$\frac{\bar{\delta}S_{(n)}(x,x';t_{n-1},\ldots,t_1)}{\bar{\delta}x^{\mu-}} = -\frac{1}{N_c}\langle\overline{\psi}(x')U(x',t_{n-1}) \times U(t_{n-1},t_{n-2})\ldots\frac{\bar{\delta}U(t_1,x)}{\bar{\delta}x^{\mu-}}\psi(x)\rangle. \qquad (8)$$

3. Integrodifferential equation

The above Green's functions satisfy the following equations of motion concerning the quark field variables:

$$(i\gamma.\partial_{(x)} - m)S_{(n)}(x,x';t_{n-1},\ldots,t_1) = i\delta^4(x-x') \times e^{F_n(x,t_{n-1},\ldots,t_1)} + i\gamma^\mu\frac{\bar{\delta}S_{(n)}(x,x';t_{n-1},\ldots,t_1)}{\bar{\delta}x^{\mu-}}, \qquad (9)$$

which become for $n = 1$

$$(i\gamma.\partial_{(x)} - m)\,S(x,x') = i\delta^4(x-x') + i\gamma^\mu\frac{\bar{\delta}S(x,x')}{\bar{\delta}x^{\mu-}}. \qquad (10)$$