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Distribution of Angular Momentum in the Transverse Plane

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Abstract

Fourier transforms of GPDs describe the distribution of partons in the transverse plane. The 2nd moment of GPDs has been identified by X.Ji with the angular momentum (orbital plus spin) carried by the quarks - a fundamental result that is being widely utilized in the spin decomposition of a longitudinally polarized nucleon. However, we will demonstrate that, despite the above results, the Fourier transform of the 2nd moment of GPDs does not describe the distribution of angular momentum in the transverse plane for a longitudinally polarized target.

Keywords: GPDs, angular momentum

1. Introduction

The 2-dimensional Fourier transform of the Generalized Parton Distribution (GPD) $H(x,0,t)$ yields the distribution of partons in the transverse plane for an unpolarized target [1].

$$q(x,\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} H(x,0,-\vec{\Delta}_\perp^2)\, e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \tag{1}$$

As a corollary, one finds that the distribution of charge in the transverse plane is given by the 2-dimensional Fourier transform of the Dirac form factor $F_1(t)$ [2]. These results are relativistically correct in contradistinction to the interpretation of the 3-dimensional Fourier transform of form factors as charge distributions in 3-dimensional space.

GPDs can also be used to determine the angular momentum carried by quarks of flavor q using the Ji-relation [3]

$$J_q = \frac{1}{2}\int dx\, x\left[H_q(x,\xi,0) + E_q(x,\xi,0)\right], \tag{2}$$

which requires GPDs extrapolated to momentum transfer $t = 0$.In Ref.[4], for the chiral quark soliton model,a relation between the 3-dimensional Fourier transform of

$$J_q(t) \equiv \frac{1}{2}\int dx\, x\left[H_q(x,\xi,t) + E_q(x,\xi,t)\right] \tag{3}$$

and the distribution of angular momentum in position space was suggested. While Lorentz invariance is not an issue for the model considered in Ref.[4], in general the interpretation of the 3-dimensional Fourier transform of a generalized form factor as a distribution in 3-dimensional space is inconsistent with Lorentz invariance [1, 2]. However, since the interpretation of the 2-dimensional Fourier transform of generalized form factors as distributions in the transverse plane (in the infinite momentum frame) is relativistically correct, it is frequently cited that the distribution of angular momentum in the transverse plane can be related to the 2-dimensional Fourier transform of (3) (see e.g. [5]).

In this note, we wil thus investigate whether the 2-dimensional Fourier transform of $J_q(t)$ can be interpreted as the distribution of angular momentum in the transverse plane. Using a scalar diquark model, we will calculate the distribution of quark Orbital Angular Momentum (OAM) using two complementary approaches: in the first approach, we take the 2-dimensional Fourier transform of $J_q(t)$ calculated in this model. From that we subtract the spin-distribution in the transverse plane evaluated from the same light-cone wave functions that were used to calculate the GPDs. In the second approach we calculate the distribution of quark OAM as

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a function of the impact parameter also directly from the same light-cone wave functions used in the first approach.

We selected the scalar diquark model for this study not because we think it is a good approximation for QCD, but to make a point of principle for which that fact that it is straightforward to maintain Lorentz invariance in this model is very important. Furthermore, since it is not a gauge theory, no issues arise as to whether one should include the vector potential in the definition of OAM or in which gauge the calculation should be done, i.e. there is no difference between Ji's OAM (2) and that of Jaffe and Manohar [6].

2. Distribution of Angular Momentum in the Transverse Plane

Following Ref. [4], we define

$$\tilde{J}(\vec{b}_\perp) \equiv \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} J_q(-\vec{\Delta}_\perp^2), \tag{4}$$

where

$$\begin{aligned} J_q(-\vec{\Delta}_\perp^2) &\equiv \frac{1}{2}\int dx x[H_q(x,\xi,-\vec{\Delta}_\perp^2) + E_q(x,\xi,-\vec{\Delta}_\perp^2)] \\ &\equiv \frac{1}{2}[A_q(-\vec{\Delta}_\perp^2) + B_q(-\vec{\Delta}_\perp^2)]. \end{aligned} \tag{5}$$

The main goal of this work is to investigate whether it is justified to interpret $\tilde{J}(\vec{b}_\perp)$ as the distribution of angular momentum in the transverse plane.

Calculating the relevant GPDs is straightforward using the light-cone wave functions [7] for the scalar diquark model

$$\begin{aligned} \psi^\uparrow_{+\frac{1}{2}}\left(x,\vec{k}_\perp\right) &= \left(M+\frac{m}{x}\right)\phi(x,\vec{k}_\perp^2) \\ \psi^\uparrow_{-\frac{1}{2}}(x,\vec{k}_\perp) &= -\frac{k^1+ik^2}{x}\phi(x,\vec{k}_\perp^2) \\ \psi^\downarrow_{+\frac{1}{2}}(x,\vec{k}_\perp) &= \frac{k^1+ik^2}{x}\phi(x,\vec{k}_\perp^2), \\ \psi^\downarrow_{-\frac{1}{2}}(x,\vec{k}_\perp) &= (M+\frac{m}{x})\phi(x,\vec{k}_\perp^2) \end{aligned} \tag{6}$$

with $\phi(x,\vec{k}_\perp^2) = \frac{g/\sqrt{1-x}}{M^2-\frac{\vec{k}_\perp^2+m^2}{x}-\frac{\vec{k}_\perp^2+\lambda^2}{1-x}}$. Here g is the Yukawa coupling and $M/m/\lambda$ are the masses of the 'nucleon'/'quark'/diquark respectively. Furthermore x is the momentum fraction carried by the quark and $\vec{k}_\perp \equiv \vec{k}_{\perp e} - \vec{k}_{\perp\gamma}$ represents the relative $\perp$ momentum. The upper wave function index $\uparrow$ refers to the helicity of the 'nucleon' and the lower index to that of the quark.

For the generalized form factors needed to evaluate (5) one finds [7]

$$A_q(-\vec{\Delta}_\perp^2) = \int dx\, xH_q(x,0,-\vec{\Delta}_\perp^2) \tag{7}$$

where

$$\begin{aligned} H_q(x,0,-\vec{\Delta}_\perp^2) = \int \frac{d^2\vec{k}_\perp}{16\pi^3}\Big[&\psi^{\uparrow *}_{+\frac{1}{2}}(x,\vec{k}'_\perp)\psi^\uparrow_{+\frac{1}{2}}(x,\vec{k}_\perp) \\ &+ \psi^{\uparrow *}_{-\frac{1}{2}}(x,\vec{k}'_\perp)\psi^\uparrow_{-\frac{1}{2}}(x,\vec{k}_\perp)\Big] \end{aligned} \tag{8}$$

where $\vec{k}'_\perp = \vec{k}_\perp + (1-x)\vec{\Delta}_\perp$ as well as

$$B_q(-\vec{\Delta}_\perp^2) = \int dx\, xE(x,0,-\vec{\Delta}_\perp^2) \tag{9}$$

$$\begin{aligned} E_q(x,0,-\vec{\Delta}_\perp^2) = \frac{-2M}{\Delta^1 - i\Delta^2}\int\frac{d^2\vec{k}_\perp}{16\pi^3}\Big[&\psi^{\uparrow *}_{+\frac{1}{2}}(x,\vec{k}'_\perp)\psi^\downarrow_{+\frac{1}{2}}(x,\vec{k}_\perp) \\ &+ \psi^{\uparrow *}_{-\frac{1}{2}}(x,\vec{k}'_\perp)\psi^\downarrow_{-\frac{1}{2}}(x,\vec{k}_\perp)\Big]. \end{aligned} \tag{10}$$

From these GPDs one can determine the OAM as obtained from GPDs through the Ji relation (2) as

$$L_q = \frac{1}{2}\int_0^1 dx\left[xH_q(x,0,0) + xE(x,0,0) - \Delta q(x)\right], \tag{11}$$

where

$$\Delta q(x) = \int\frac{d^2\vec{k}_\perp}{16\pi^3}\left[\left|\psi^\uparrow_{+\frac{1}{2}}(x,\vec{k}_\perp)\right|^2 - \left|\psi^\uparrow_{-\frac{1}{2}}(x,\vec{k}_\perp)\right|^2\right]. \tag{12}$$

Since some of the above $\vec{k}_\perp$-integrals diverge, a manifestly Lorentz invariant Pauli-Villars regularization (subtraction with heavy scalar $\lambda^2 \to \Lambda^2$) is always understood.

To evalulate relation (4), we simplify and rewrite (8) and (10) as:

$$\begin{aligned} H(x,0,-\vec{\Delta}_\perp^2) = \frac{g^2}{16\pi^3}\int d^2\vec{k}_\perp \\ \Bigg[\int_0^1 \frac{d\alpha(1-x)(m+xM)^2}{[(\vec{k}_\perp+(1-x)\vec{\Delta}_\perp\,\alpha)^2+F]^2} \\ + \frac{1-x}{2(\vec{k'}^2_\perp + u)} + \frac{1-x}{2(\vec{k}_\perp^2+u)} \\ - \int_0^1 d\alpha\,\frac{(1-x)(u+\frac{(1-x)^2\vec{\Delta}_\perp^2}{2})}{((\vec{k}_\perp+(1-x)\vec{\Delta}_\perp\,\alpha)^2+F)^2}\Bigg] \end{aligned} \tag{13}$$

where
$u = x^2 - 2x + 1 + x\lambda^2$ and
$F = (1-x)^2\vec{\Delta}_\perp^2\,\alpha(1-\alpha) + x^2 - 2x + 1 + x\lambda^2$