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Ab initio nonperturbative calculations in Yukawa model

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Abstract

We give a brief review of a nonperturbative approach to field theory based on the decomposition of the state vector in Fock components, and on the covariant formulation of light-front dynamics, together with the Fock sector dependent renormalization scheme. The approach is applied to the calculation, in the three-body Fock space truncation, of electromagnetic form factors of a fermion in the Yukawa model (in particular, of anomalous magnetic moment). Once the renormalization conditions are properly taken into account, the anomalous magnetic moment does not depend on the regularization scale, when the latter is much larger than the physical masses.

Keywords: Light-front dynamics, Fock sectors, electromagnetic form factors

1. Introduction

In the quantum field theory, due to the particle creation and annihilation, the number of particles in a system is not fixed and the state vector is a superposition of the states with different numbers of particles:

$$|p\rangle = \sum_{n=1}^{\infty} \int \psi_n(k_1, \ldots, k_n, p)\,|n\rangle\, D_k. \tag{1}$$

ψ_n is the n-body wave function and D_k is the integration measure which we don't precise here. This decomposition can be represented by the Fock column each line of which corresponds to a given number of particles and contains the Fock component ψ_n. In the cases when we can expect that the decomposition (1) converges enough quickly, we can make truncation, that is replace the infinite sum in (1) by the finite one. Then, substituting truncated state vector in the eigenvector equation

$$H\,|p\rangle = M\,|p\rangle\,,$$

we obtain a finite coupled system of equations for the Fock components ψ_n which can be solved numerically. To completely determine the state vector, we normalize it according to

$$\langle p'|p\rangle = 2p_0\delta^{(3)}(\mathbf{p}' - \mathbf{p}). \tag{2}$$

With the decomposition (1), the normalization condition (2) writes

$$\sum_{n=1}^{\infty} I_n = 1, \tag{3}$$

where I_n is the contribution of the n-body Fock sector to the norm.

In this way we do not require the smallness of the coupling constant. The approximate (truncated) solution is non-perturbative. This is the basis of non-perturbative approach which we developed in a series of papers [1, 2, 3, 4, 5] (see for review [6]) and which will be briefly reviewed in the present contribution.

The main difficulty on this way is to ensure cancellation of infinities. In perturbative approach, for a renormalizable field theory, the cancellation of infinities is obtained as a by-product after renormalization in any fixed order of coupling constant. For this cancelation it is important to take into account full set of graphs in a given order. Omitting some of these graphs destroys the cancellation and the infinities survive after renormalization. Namely that happens after truncation: the truncated solution includes all the orders of the coupling constant, but not full set of graphs in any given order. Therefore the standard renormalization scheme

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does not eliminate infinities. To provide cancellation of infinities, there was proposed [7] the sector-dependent renormalization scheme. This scheme, in which the values of the counter terms are precised from sector to sector according to unambiguously formulated rules, was developed in detail in [3] and applied to calculation of observables in [4, 5]. Below we will give its brief review and show some results obtained in this approach.

In sect. 2 we discuss the convergence of the decomposition (1). In sect. 3 the system of equation for the Fock components is presented. The sector dependent renormalization scheme is briefly described in sect. 4. In sect. 5 we explain how to calculate the electromagnetic form factors. The numerical results are given in sect. 6 and conclusions are presented in sect. 7.

2. On convergence of the Fock decomposition

We work in the light-front dynamics (LFD) [8, 9, 10], more precisely, in its explicitly covariant version [8, 9]. In four-dimensional space, the state vector (1) is defined on the light-front plane of general orientation $\omega{\cdot}x = 0$, where ω is an arbitrary four-vector restricted by the condition $\omega^2 = 0$ [8, 9]. The traditional form of LFD [10] is recovered by using $\omega = (1, 0, 0, -1)$.

As mentioned, the truncation of the Fock decomposition can be efficient if this decomposition (1) converges enough quickly. The convergence depends, of course, on the nature of system under consideration. If this system is dominated by a finite number of degrees of freedom (like hadrons in quark models), then the decomposition (1) is determined with good accuracy by finite number of the components. Notice that these "degrees of freedom", e.g. quarks as basis of decomposition (1), may be some efficient, dressed constituents.

The convergence of the Fock decomposition was estimated [11] in the explicitly solvable Wick-Cutkosky model [12]. This model corresponds to spineless massive particles with equal masses m interacting by spineless massless exchange. One can find the two-body Bethe-Salpeter amplitude. The requirement for the electromagnetic form factor $F(Q^2 = 0) = 1$ fixes the normalization of the Bethe-Salpeter amplitude. On the other hand, projecting the Bethe-Salpeter amplitude on the light-front plane, we find the two-body Fock component of the state vector (1). Its normalization integral is not 1 but gives the two-body contribution to full normalization. One can also estimate the valence three-body contribution. We chose the parameters maximally unfavorable for dominance of a few-body sector. Namely, the coupling constant is very strong: $\alpha = 2\pi$ that corresponds to the limiting case when binding energy in Wick-Cutkosky model $E_b = -2m$ compensates full mass of the system. Strong coupling constant increases contribution of higher orders, i.e., of many-body components. In addition, since the exchange particles are massless, they can be easy created. The result for different contributions is given in the table 1 taken from ref. [11]. We see that even in this unfavorable case the

N_2	N_3	$N_{n\geq4}$	$N_2 + N_3 + N_{n\geq4}$
0.643	0.257	0.100	1

Table 1: Contributions of the Fock sectors with the particle numbers $n = 2$, $n = 3$ and $n \geq 4$ ($N_{n\geq4} = \sum_{n=4}^{\infty} N_n$) to the full normalization integral $N = \sum_{n=2}^{\infty} N_n = 1$ of the state vector for $M = 0$ ($\alpha = 2\pi$).

Fock states with 2 and 3 particles contribute 90% in the normalization integral. This would give 10% accuracy in calculation of observables, say, the electromagnetic form factor.

3. System of equations

We study Yukawa model containing the fermion field ψ and the scalar field ϕ with the interaction vertex $g_0\bar{\psi}\psi\phi$. For regularization, we include in the Lagrangian the Pauli-Villars fields (one fermion and one boson), which also appear in the basis of decomposition (1). Since our formalism is explicitly covariant, the spin structure of the wave function ψ_n is very simple. It should incorporate however ω-dependent components. Therefore the spin structure of the two-body component in the Yukawa model reads:

$$\bar{u}(k_1)\Gamma_2 u(p) = \bar{u}(k_1)\left[b_1 + \frac{M\, \not{\omega}}{\omega{\cdot}p}\, b_2\right]u(p). \tag{4}$$

where $\not{\omega} = \omega_\mu\gamma^\mu$. The coefficients b_1 and b_2 are scalar functions determined by dynamics. In LFD they depend on well-known variables $k_\perp, x$: $b_{1,2} = b_{1,2}(k_\perp, x)$.

System of equations for the Fock components in the truncation $N = 3$ is graphically shown in fig. 1. One can exclude the three-body component and obtain reduced system of equations which includes the one- and two-body components only. It is shown in fig. 2. The one-body components is a constant which is determined from the normalization condition. Therefore what was solved numerically was the reduced system fig. 2.

We remind that the Pauli-Villars particles are included in the Lagrangian and appear in the basis of