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Running cosmological constant with observational tests

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ABSTRACT

We investigate the running cosmological constant model with dark energy linearly proportional to the Hubble parameter, $\Lambda = \sigma H + \Lambda_0$, in which the ΛCDM limit is recovered by taking $\sigma = 0$. We derive the linear perturbation equations of gravity under the Friedmann–Lemaître–Robertson–Walker cosmology, and show the power spectra of the CMB temperature and matter density distribution. By using the Markov chain Monte Carlo method, we fit the model to the current observational data and find that $\sigma H_0/\Lambda_0 \lesssim 2.63 \times 10^{-2}$ and 6.74×10^{-2} for $\Lambda(t)$ coupled to matter and radiation-matter, respectively, along with constraints on other cosmological parameters.

1. Introduction

The type-Ia supernova observations [1,2] have shown that our universe is undergoing a late-time accelerating expansion, which is caused by Dark Energy [3]. The simplest way to realize such a late-time accelerating mechanism is to introduce a cosmological constant to the gravitational theory, such as that in the ΛCDM model. This model fits current cosmological observations very well, but there exist several difficulties, such as the "fine-tuning" [4,5] and "coincidence" [6] problems.

In this work, we will concentrate on the latter problem [7], which has been extensively explored in the literature. One of the popular attempts is the running Λ model, in which the cosmological constant evolves in time and decays to matter in the evolution of the universe [8–20], so that the present energy densities of dark energy and dark matter are of the same order of magnitude. Its observational applications have been investigated in Refs. [21–23]. In our study, we are interested in the specific model with $\Lambda = \sigma H$ [24–29], which would originate from the theory with the QCD vacuum condensation associated with the chiral phase transition [30–34]. In this scenario, the cosmological constant decays to matter (non-relativistic) and radiation (relativistic), leading to a large number of particles created in the cosmological evolution. Without loss of generality, we phenomenologically extend this model to include that Λ additionally couples to radiation with $\Lambda = \sigma H + \Lambda_0$ [35–37], in which the ΛCDM limit can be realized if $\sigma = 0$. In this scenario, when dark energy dominates the universe, the decay rate of Λ is reduced, and the late-time accelerating phase occurs, describing perfectly the evolution history of the universe. As a result, it is reasonable to go further to analyze the cosmological behavior of this model at the sub-horizon scale.

In this paper, we examine the matter power spectrum $P(k)$ and CMB temperature perturbations in the linear perturbation theory of gravity. By using the Markov chain Monte Carlo (MCMC) method, we perform the global fit from the current observational data and constrain the model.

This paper is organized as follows: In Sec. 2, we introduce the $\Lambda(t)$CDM model and review its background cosmological evolutions. In Sec. 3, we calculate the linear perturbation theory and illustrate the power spectra of the matter distribution and CMB temperature by the **CAMB** program [38]. In Sec. 4, we use the **CosmoMC** package [39] to fit the model from the observational data and show the constraints on cosmological parameters. Our conclusions are presented in Sec. 5.

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2. The running cosmological constant model

We start with the Einstein equation, given by

$$R_{\mu\nu} - \frac{g_{\mu\nu}}{2} R + \Lambda(t) g_{\mu\nu} = \kappa^2 T^M_{\mu\nu} \,, \tag{1}$$

where $\kappa^2 = 8\pi G$, $R = g^{\mu\nu} R_{\mu\nu}$ is the Ricci scalar, $\Lambda(t)$ is the time-dependent cosmological constant, and $T^M_{\mu\nu}$ is the energy–momentum tensor of matter and radiation. In the Friedmann–Lemaître–Robertson–Walker (FLRW) case,

$$ds^2 = a^2(\tau)\left[-d\tau^2 + \delta_{ij} dx^i dx^j\right] \,, \tag{2}$$

we obtain

$$H^2 = \frac{a^2\kappa^2}{3}\left(\rho_M + \rho_\Lambda\right) \,, \tag{3}$$

$$\dot{H} = -\frac{a^2\kappa^2}{2}\left(\rho_M + P_M + \rho_\Lambda + P_\Lambda\right) \,, \tag{4}$$

where τ is the conformal time, $H = da/(ad\tau)$ represents the Hubble parameter, ρ_M (P_M) corresponds to the energy density (pressure) of matter and radiation, and ρ_Λ (P_Λ) is the energy density (pressure) of the cosmological constant. We note that from the relation of $\rho_\Lambda = -P_\Lambda = \kappa^{-2}\Lambda(t)$, derived from Eq. (1), one has the equation of state (EoS) of Λ to be

$$w_\Lambda \equiv \frac{P_\Lambda}{\rho_\Lambda} = -1 \,. \tag{5}$$

In Eq. (1), we consider $\Lambda(t)$ to be a linear function of the Hubble parameter, given by [29,35–37]

$$\Lambda = \sigma H + \Lambda_0 \,, \tag{6}$$

where σ and Λ_0 are two free parameters. From Eq. (6), we can write ρ_Λ with two dimensionless parameters $\lambda_{0,1}$ as

$$\rho_\Lambda = \rho^0_\Lambda\left[\lambda_0 + \lambda_1\left(\frac{H}{H_0}\right)\right] \,, \tag{7}$$

where $\rho^0_\Lambda \equiv \rho_\Lambda|_{z=0}$ is the current dark energy density with the condition $\lambda_0 + \lambda_1 = 1$ and $\lambda_1 = \sigma H_0/(\sigma H_0 + \Lambda_0)$. Note that λ_0 has been treated as a constant of integration and set to zero in Ref. [29]. Without loss of generality, we will keep λ_0 as a free parameter with the ΛCDM model recovered when $\lambda_0 \to 1$.

Substituting Eq. (7) into the conservation equation $\nabla^\mu(T^M_{\mu\nu} + T^\Lambda_{\mu\nu}) = 0$, we have

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda)\rho_\Lambda = \dot{\rho}_\Lambda \propto \dot{H} \neq 0 \,, \tag{8}$$

resulting in that dark energy unavoidably couples to matter and radiation, given by

$$\dot{\rho}_m + 3H\rho_m = Q_m \,, \tag{9}$$

$$\dot{\rho}_r + 4H\rho_r = Q_r \,, \tag{10}$$

where $Q_{m,r}$ are the decay rate from $\Lambda(t)$ to matter and radiation, taken to be

$$Q_i = \frac{\dot{\rho}_\Lambda C_i(\rho_i + P_i)}{\sum_{j=m,r} C_j(\rho_j + P_j)} \,, \tag{11}$$

respectively. Note that the analytical solution of Eq. (8) has been obtained with $\lambda_0 = 0$ and $w_M = constant$ in Refs. [24,25]. However, if $\lambda_0 \neq 0$ and $\rho_M = \rho_m + \rho_r$, composited of multi-fluid with EoS $w_r \neq w_m$, the analytical solution no longer exists.

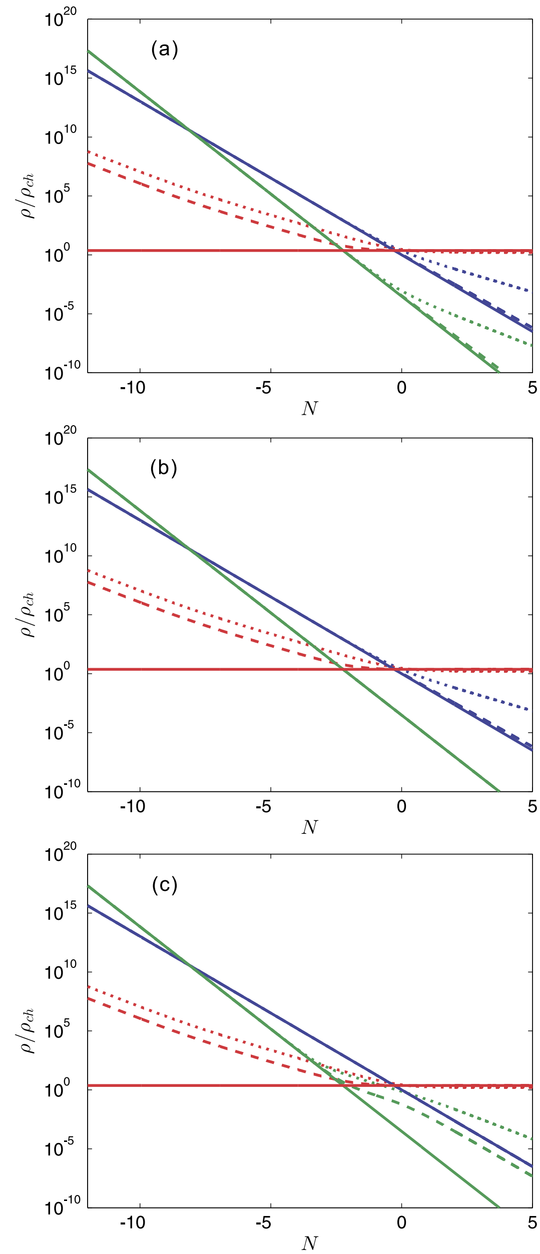

Fig. 1. Evolutions of ρ_m (blue line), ρ_r (green line) and ρ_Λ (red line) with (a), (b) and (c) corresponding to $(C_r, C_m) = (1, 1)$, $(0, 1)$ and $(1, 0)$, where the solid, dashed and dotted lines represent $(\lambda_0, \lambda_1) = (1, 0)$, $(0.9, 0.1)$ and $(0, 1)$, respectively. The initial conditions are taken as $\rho_m a^3/\rho_{ch} = 1$ and $\rho_r a^4/\rho_{ch} = 3 \times 10^{-4}$ at $N \equiv \ln a = -12$, where ρ_{ch} is the characteristic energy density. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

In Fig. 1, we show the cosmological evolutions of ρ_m (blue line), ρ_r (green line) and ρ_Λ (red line), normalized by the characteristic energy density ρ_{ch}, as functions of the e-folding $N \equiv \ln a$ with $\rho_m a^3/\rho_{ch} = 1$ and $\rho_r a^4/\rho_{ch} = 3 \times 10^{-4}$ at $N = -12$, where (C_r, C_m) are (a) $(1, 1)$, (b) $(1, 0)$ and (c) $(0, 1)$ with $(\lambda_0, \lambda_1) = (1, 0)$, $(0.9, 0.1)$ and $(0, 1)$, corresponding to the solid, dashed and dotted lines, respectively. In Fig. 1c, we observe that if dark energy fully decays to radiation, ρ_r can be of the same order of ρ_m at $\lambda_1 \gtrsim 0.1$, which violates the current observations. On the contrary, this problem never occurs if $\Lambda(t)$ only couples to matter as shown in Fig. 1b. This behavior allows us to fix $C_m = 1$ and keep C_r to be a free parameter in the later study. In Fig. 2, we present $a^3\rho_m$ (blue